Linking Corrections to Extremality and Entropy

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work w/
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Based On [1909.05254]

$$\Delta M \approx -T\Delta S$$

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How do thermodynamic quantities react to a (small) change in the theory?

1. Take system with dynamics $\mathcal{L}_0$
2. Change the dynamics: $\mathcal{L}_0 \longrightarrow \mathcal{L}_0 + \Delta\mathcal{L}_0$
3. How do quantities like **Entropy** (S) or **Energy** (M) change?

Focus: $\Delta M_{\rm ext}(\vec{\mathcal{Q}})$ and $\Delta S(M, \vec{\mathcal{Q}})$

Take a system specified by energy M and other parameters $\vec{\mathcal{Q}}$

Alter the dynamics, perturbatively

Question 1: how does the entropy of state $(M, \vec{\mathcal{Q}})$ change?

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"Extremality bound"

$$\lim_{T \to 0} M(\vec{\mathcal{Q}}, T) \equiv M_{\rm ext}(\vec{\mathcal{Q}}) \leq M(\vec{\mathcal{Q}}, T)$$

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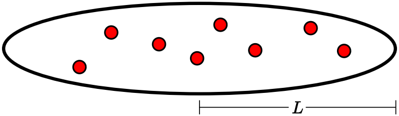
Will Show: ΔS and $\Delta M_{\rm ext}$ are directly linked

Basic Example: Ideal Gas

- Classical 2D ideal gas w/ N particles: $H = \sum_i \frac{\vec{p}_i^2}{2m}$. ($\vec{\mathcal{Q}} = N$)



- Basic thermodynamic relations give:

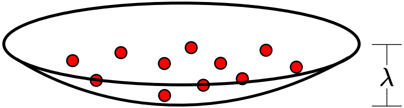
$$M_0(N,T) = NT \qquad S_0(N,T) \approx N\left(\ln\left(2\pi T m L^2\right) + 2\right)$$

- Clearly, energy is bounded by zero:

$$M(N,T) > M^0_{\text{ext}}(N) = 0$$

Basic Example: Ideal Gas $\Delta M_{\rm ext}(N)$

Curve the disk: $H \longrightarrow H - \lambda\left(1 - \frac{r^2}{L^2}\right)$



All particles simply sitting in bottom of well at $T = 0$

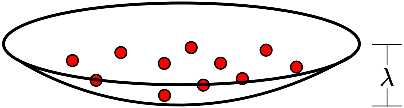
Minimum energy is shifted by $-N\lambda$:

$$M(N,T) > M^0_{\rm ext}(N) + \Delta M_{\rm ext}(N) \qquad \Delta M_{\rm ext}(N) = -N\lambda$$

Basic Example: Ideal Gas $\Delta S(M,N)$

Next find T and $\Delta S(M,N)$

Consider a state which is slightly perturbed, $M \gg N\lambda$



Corrected temperature and entropy are:

$$T(M,N) = \frac{M}{N}\left(1 + \frac{N\lambda}{2M} + \dots\right) \quad \Delta S(M,N) = \frac{N^2\lambda}{2M}\left(1 - \frac{N\lambda}{6M} + \dots\right)$$

Inspecting above and $\Delta M_{\rm ext}(N) = -N\lambda$, we find

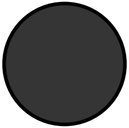
$$-T(M,N)\Delta S(M,N) = \Delta M_{\rm ext}(N)\left(1 + \frac{N\lambda}{3M} + \dots\right)$$

Relations of this form completely generic

Goals of the Talk

1. Argue that $\Delta M \approx -T\Delta S$ generically

2. Convince you that there are more interesting applications



$M, Q, J, \ldots$

Black Hole Corrections: Cheung, Remmen, Liu

[1801.08546]

- An inspiring story about BHs
- Review of charged black holes:

$$\mathcal{L}_0 = \frac{1}{16\pi}\left(R - F_{\mu\nu}^2\right) \qquad (G_N = 1)$$

$$S_0(M,Q) = \frac{\text{Area}}{4} = \pi r_+^2 \qquad T_0(M,Q) = \frac{r_+ - M}{2\pi r_+^2}$$

$$r_+ \equiv M + \sqrt{M^2 - Q^2}$$

- Examining states with charge Q, there's an extremal mass

$$M > Q \;\equiv M_{\text{ext}}^0(Q) = \lim_{T_0 \to 0} M(Q, T_0)$$

Black Hole Corrections: Cheung, Remmen, Liu

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- Now change the theory (as many have previously):

$$\Delta\mathcal{L} = c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu} + c_3 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + c_4 R F_{\mu\nu}F^{\mu\nu} + c_5 R_{\mu\nu}F^{\mu\rho}F^{\nu}{}_{\rho} + c_6 R_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma} + c_7 F_{\mu\nu}F^{\mu\nu}F_{\rho\sigma}F^{\rho\sigma} + c_8 F_{\mu\nu}F^{\nu\rho}F_{\rho\sigma}F^{\sigma\mu}.$$

- Calculations to do:
  1. Find the solution corrections: $g_{\mu\nu} \to g_{\mu\nu} + \Delta g_{\mu\nu}$, $A_\mu \to A_\mu + \Delta A_\mu$
  2. Re-compute relation between T, M, Q from near horizon metric
  3. Use result to find new extremality bound
  4. Re-calculate S from Wald's formula ($S \neq \frac{\text{Area}}{4}$). Area changed
  5. Examine results for ΔM_{ext} and ΔS

Black Hole Corrections

Findings: $\Delta M_{\rm ext}$ and $\Delta S(M, Q)$ near extremality depend on

same combination of EFT coefficients

Similar Results

1. [1801.08546] Cheung et al., holds in arbitrary dimensions for Reissner-Nordström. Metric-based understanding.
2. [1901.11535] Reall & Santos, holds for higher-derivative corrections to uncharged, 4D spinning black holes (Kerr)
3. [1903.09156] Cheung et al., holds for 4D, spinning, electrically and magnetically charged black holes (Kerr-Newman)

All results a consequence of $\Delta M_{\rm ext} \approx -T\Delta S$

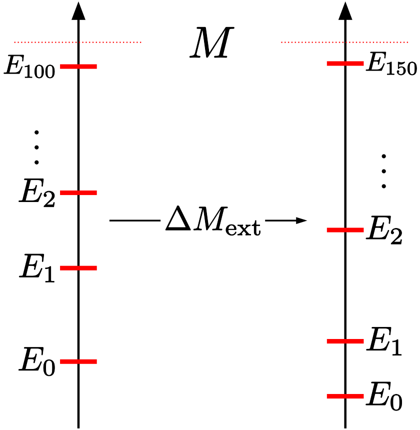
Proving

$$\Delta M_{\rm ext} \approx -T\Delta S$$

Heuristic Argument for $\Delta M_{\rm ext} \propto -\Delta S$

Technical steps straightforward,
but first a plausibility argument

- Say $\Delta M_{\rm ext} < 0$
- Roughly, microscopic energy levels of system shift down
- ΔS is change at fixed M
- Downward shift in energies $\implies$ more ways of making state M, $\Delta S > 0$
- Opposite for $\Delta M_{\rm ext} > 0$



Technical Argument: Setup

Consider system with extensive variables S, M, Q

Let the free energy $G(T, \mu) = M - TS - \mu Q$ take the form

$$\boxed{G(T, \mu) = G_0(T, \mu) + \Delta G(T, \mu)}$$

As usual, $S(\mu, T)$ and $Q(\mu, T)$ are given by

$$S(\mu, T) = -\left(\frac{\partial G}{\partial T}\right)_\mu \qquad Q(\mu, T) = -\left(\frac{\partial G}{\partial \mu}\right)_T$$

From $G(\mu, T)$ we can compute both $\Delta M_{\rm ext}(Q)$ and $\Delta S(M, Q)$

Technical Argument: $\Delta M_{\rm ext}(Q)$

Given G, the energy is (by definition):

$$M(\mu,T) = G_0(\mu,T) + \Delta G(\mu,T) + T\,S(\mu,T) + \mu\,Q(\mu,T)$$

We want $\boxed{M(Q,T)}$, since $M_{\rm ext}(Q) = \lim_{T\to 0} M(Q,T)$.

Strategy: Get $Q(\mu,T)$ from G, invert to find $\mu(Q,T)$ and form

$$M(Q,T) = M(\mu,T)\Big|_{\mu=\mu(Q,T)}$$

Technical Argument: $\Delta M_{\rm ext}(Q)$

Straightforward steps:

1. Solve $Q(\mu,T)=-\frac{\partial G_0}{\partial\mu}-\frac{\partial\Delta G}{\partial\mu}$ to find $\mu(Q,T)$
2. Results in $\mu(Q,T)=\mu_0(Q,T)+\Delta\mu(Q,T)$. In particular

$$-\left(\frac{\partial G_0(\mu,T)}{\partial\mu}\right)_T\Big|_{\mu=\mu_0(Q,T)}=Q$$

3. Then $\mu\to\mu(Q,T)$ everywhere. Final result (to 1st order):

$$M(Q,T)=G_0(\mu_0,T)+\Delta G(\mu_0,T)+\mu_0 Q+T\,S$$

Take the $T\to 0$ limit and use 3rd law: $\lim_{T\to 0}TS(T,Q)=0$

$$M^0_{\rm ext}(Q)=\lim_{T\to 0}G_0(\mu_0,T)+\mu_0 Q \qquad \boxed{\Delta M_{\rm ext}(Q)=\lim_{T\to 0}\Delta G(\mu_0(Q,T),T)}$$

Technical Argument: $\Delta S(M,Q)$

Reall & Santos [1901.11535] related $\Delta S(M,Q)$ and $\Delta G(\mu,T)$

Just the chain rule and thermo. identities

Write $G(\mu,T,\epsilon)=G_0(\mu,T)+\epsilon\Delta G(\mu,T)$, to track orders

Similarly, we have $S=S(M,Q,\epsilon)$

Quantities of interest:

$$\Delta G(\mu,T)=\left(\frac{\partial G(\mu,T,\epsilon)}{\partial\epsilon}\right)_{\mu,T}\Big|_{\epsilon=0}$$

$$\Delta S(M,Q)=\left(\frac{\partial S(M,Q,\epsilon)}{\partial\epsilon}\right)_{M,Q}\Big|_{\epsilon=0}$$

Technical Argument: $\Delta S(M,Q)$

Step 1: Compute $\Delta G(\mu,T)$ explicitly using $G = M - TS - \mu Q$

$$\beta\Delta G(\mu,T) = \left(\frac{\partial \beta G}{\partial \epsilon}\right)_{\mu,T} = \beta\left(\frac{\partial M}{\partial \epsilon}\right)_{\mu,T} - \mu\left(\frac{\partial Q}{\partial \epsilon}\right)_{\mu,T} - \left(\frac{\partial S}{\partial \epsilon}\right)_{\mu,T}$$

Above, M, Q, S are functions of μ, T, ϵ

Step 2: Relate to $\Delta S(M,Q) = \left(\frac{\partial S(M,Q,\epsilon)}{\partial \epsilon}\right)_{M,Q}$ via chain rule.

Key is that $S(\mu,T,\epsilon) = S(M(\mu,T,\epsilon), Q(\mu,T,\epsilon),\epsilon)$

$$\begin{aligned}\left(\frac{\partial S}{\partial \epsilon}\right)_{\mu,T} &= \left(\frac{\partial S}{\partial M}\right)_{Q,\epsilon}\left(\frac{\partial M}{\partial \epsilon}\right)_{\mu,T} + \left(\frac{\partial S}{\partial Q}\right)_{M,\epsilon}\left(\frac{\partial Q}{\partial \epsilon}\right)_{\mu,T} + \left(\frac{\partial S}{\partial \epsilon}\right)_{M,Q} \\ &= \beta\left(\frac{\partial M}{\partial \epsilon}\right)_{\mu,T} - \beta\mu\left(\frac{\partial Q}{\partial \epsilon}\right)_{\mu,T} + \Delta S(M,Q) \qquad (1^{\text{st}}\ \text{Law})\end{aligned}$$

Step 3: Substitute: $\boxed{\Delta S(M,Q) = -\frac{1}{T}\Delta G(\mu,T)}$

Technical Argument: Relating $\Delta M_{\rm ext}$ and ΔS

Two Results: both depend on the same function $\Delta G(\mu, T)$

$$\Delta M_{\rm ext}(Q) = \lim_{T\to 0} \Delta G\left(\mu_0(Q,T), T\right)$$
$$-T(M,Q)\Delta S(M,Q) = \Delta G\left(\mu_0(M,Q), T_0(M,Q)\right)$$

Must evaluate second at some $M(Q)$ and compare

Two Cases: Value of $M(Q)$ depends on sign of $\Delta M_{\rm ext}$

Technical Argument: Relating $\Delta M_{\rm ext}$ and ΔS

$$\Delta M_{\rm ext}(Q) = \lim_{T\to 0} \Delta G\left(\mu_0(Q,T),T\right)$$

$$-T(M,Q)\Delta S(M,Q) = \Delta G\left(\mu_0(M,Q),T_0(M,Q)\right)$$

Case 1: if $\Delta M_{\rm ext}(Q) < 0$, state with $M = M^0_{\rm ext}(Q)$ still exists

$$\Delta M_{\rm ext}(Q) = -T\Delta S(M^0_{\rm ext}(Q),Q) \text{ exactly}$$

Case 2: if $\Delta M_{\rm ext}(Q) > 0$, state with $M = M^0_{\rm ext}(Q)$ doesn't exist

Compute at $M = M^0_{\rm ext}(Q)\,(1+\delta)$, $\delta > 0$ such that $T_0 \gg \Delta T$

Then, relation holds approximately

$$\Delta M_{\rm ext}(Q) = -T(M,Q)\Delta S(M,Q)\Big|_{M\approx M^0_{\rm ext}(Q)} \times \left(1+\mathcal{O}\left(\delta\right)\right)$$

Completely General

Holds for arbitrary $\vec{\mathcal{Q}}$ in arbitrary ensembles

- Break $\vec{\mathcal{Q}}$ into fluctuating $\vec{\mathrm{Q}}$ and fixed $\vec{q}$
- Pass from $M(S, \vec{\mathrm{Q}}, \vec{q}) \longrightarrow G(T, \vec{\mu}, \vec{q})$, $\vec{\mu}$ conjugate to $\vec{\mathrm{Q}}$
- Proof goes through the same:

$$\boxed{\Delta M_{\rm ext}(\vec{\mathcal{Q}}) \approx -T\Delta S(M, \vec{\mathcal{Q}})\Big|_{M\approx M_{\rm ext}(\vec{\mathcal{Q}})}}$$

- No explicit dependence on dimensionality, matter fields, etc.
- We'll apply to black holes, but not special to this case

Black Hole Examples

Charged AdS_4 Black Holes

Change to theory doesn't have to be higher-derivative

$$I = \frac{1}{16\pi}\int \mathrm{d}^4x\sqrt{-g}\left(R - F^2 + \boxed{\frac{6}{\ell^2}}\right)$$

Small correction if $M \approx Q \ll \ell$. Solutions known explicitly

Old $M = Q$ extremal state no longer exists ($Q > 0$)

$$M > Q + \frac{Q^3}{2\ell^2} + \mathcal{O}\left(\frac{Q^5}{\ell^4}\right) \implies \Delta M_{\rm ext} = \frac{Q^3}{2\ell^2} > 0$$

To check: evaluate corrections at

$$M(Q) = Q(1+\delta)\ , \delta > 0$$

Choose δ such that $T_0 \gg \Delta T$ and $\Delta M_{\rm ext} = -T\Delta S$ approx.

Charged AdS_4 Black Holes

Straightforward to work everything out

$$\Delta M_{\rm ext} = \frac{Q^3}{2\ell^2}$$

$$T(M,Q)\Big|_{M=Q(1+\delta)} = \frac{\sqrt{\delta}}{\sqrt{2}\pi Q}\left(1 - \boxed{\frac{Q^2}{4\delta\ell^2}} + \ldots\right)$$

$$\Delta S(M,Q)\Big|_{M=Q(1+\delta)} = -\frac{\pi Q^{7/2}}{\sqrt{2\delta}\ell^2}\left(1 + \boxed{\frac{10\delta}{\sqrt{2}}} + \ldots\right)$$

Choose δ to lie in $1 \gg \delta \gg \frac{Q^2}{\ell^2}$. Keeps corrections small

$$\Delta M_{\rm ext}(Q) = -T(M,Q)\Delta S(M,Q)\Big|_{M=Q(1+\delta)} \times \left(1 + \mathcal{O}\left(\delta\right)\right)$$

Charged AdS_4 BHs with Higher-Derivatives

Another example: HD corrections to large $Q \gg \ell$ AdS_4 BHs

$$I_0[g_{\mu\nu}] = \frac{1}{16\pi}\int \mathrm{d}^4x\sqrt{-g}\,\left(R - F^2 + \frac{6}{\ell^2}\right)$$
$$\Delta I[g_{\mu\nu}] = \frac{-1}{16\pi}\int \mathrm{d}^4x\sqrt{-g}\,\left(\alpha_1\ell^2F^4 + \alpha_2\ell^4F^6 + \alpha_3\ell^6F^8\right)$$

Calculations are very similar, $\Delta M \approx -T\Delta S$ again

$$\Delta M_{\mathrm{ext}}(Q) = \frac{3^{1/4}Q^{3/2}}{65\ell^{1/2}}\Big(39\alpha_1 - 130\alpha_2 + 540\alpha_3\Big) \approx -T_0\Delta S(M,Q)$$

Useful Calculational Fact

The metric correction $\Delta g_{\alpha\beta}$ **isn't required** to compute $G(\mu, T)$

Because $g^0_{\alpha\beta}(\mu, T)$ **extremizes action** (w/ right bdy terms) and $G(\mu, T)$ comes from the (Euclideanized) on-shell action

$$\begin{aligned}\beta\, G(\mu, T) &= I^{\mathrm{E}}_0[g^0 + \Delta g] + \Delta I[g^0 + \Delta g] \\ &= I^{\mathrm{E}}_0[g^0] + \Delta I[g^0] + (\text{higher order})\end{aligned}$$

Using $g_{\alpha\beta}(M, Q)$ instead of $g_{\alpha\beta}(\mu, T)$, one needs $\Delta g_{\alpha\beta}(M, Q)$

Ex.: Can find thermo. corrections to extremal Kerr [1901.11535] , while $\Delta g_{\alpha\beta}(M, J)$ known only for small J [1901.01315]

Potential Implications

Intuition for ΔS

Natural setting for considering changes to dynamics: EFT

EFT thinking suggests sign for ΔS side of the relation

- Corrections to the action from UV physics, additional states
- Canonical example: Euler-Heisenberg action, integrating out $e^{\pm}$

$$\mathcal{L}_{\rm UV} = -\frac{1}{4}F^2 + \bar{\psi}\left(i\not{D} - m\right)\psi$$

$$\implies \mathcal{L}_{\rm IR} \approx -\frac{1}{4}F^2 + \boxed{\frac{e^4}{1440\pi^2 m^4}F^4 + \frac{7e^4}{5760\pi^2 m^4}\left(F\tilde{F}\right)^2 + \ldots}$$

$$= \mathcal{L}_0 + \boxed{\Delta\mathcal{L}}$$

- Intuitively, additional states $\longleftrightarrow \Delta S > 0$

Intuition for ΔS

Heuristic Reasoning: $S(M, \vec{\mathcal{Q}})$ is measure of number of microstate combinations consistent with macrostate $M, \vec{\mathcal{Q}}$ data

If adding $\Delta\mathcal{L} \longleftrightarrow$ adding UV DOF, more ways of forming $M, \vec{\mathcal{Q}}$

Cheung et al. [1801.08546] provided quantitative arguments supporting this intuition (tree-level corrections)

Kats, Motl, & Padi [0606100] briefly state conjecture of this form for corrections to black hole entropy

"We also conjecture that the first correction to the Bekenstein-Hawking entropy, arising from higher-derivative terms applied to Wald's formula, is positive in all cases. We are not aware of counterexamples" - [0606100]

Implications for $\Delta M_{\rm ext}$

If $\Delta S > 0$, then $\Delta M_{\rm ext} < 0$, due to $\Delta M_{\rm ext} = -T\Delta S$

Meaning: Comparing at fixed $\vec{\mathcal{Q}}$, perturbed $T = 0$ state is **less massive** than unperturbed $T = 0$ state

When $\vec{\mathcal{Q}}$ is single charge $Q \implies \frac{Q}{M}$ is increased at extremality

Directly connects result to Weak Gravity Conjecture

Connection to the WGC [0601001]

Weak Gravity Conjecture: Swampland-type restriction on IR physics.

The fact that $\frac{|q|}{m} > 1$ for particles like e^- is not accidental

Facet: $U(1)$ charged BHs should have $\frac{Q}{M} > 1$ at extremality

(Whereas, $\frac{Q}{M} = 1$ in unperturbed theory)

Our Work: this feature of WGC is equivalent to $\Delta S > 0$ conjecture

Also, not only true for BHs extremal due to single $U(1)$ charge

Analogous results multiple charges, spin, etc.
Extremal state less massive in corrected theory

$$\Delta M(\vec{\mathcal{Q}}) = -T(M, \vec{\mathcal{Q}})\Delta S(M, \vec{\mathcal{Q}})\Big|_{M \approx M^0_{\rm ext}(\vec{\mathcal{Q}})}$$

Conclusions

Conclusions

- $\Delta M \approx -T\Delta S$ is a universal relation
- Explains recent findings for BH corrections
- Directly connected to the Weak Gravity Conjecture ($= \Delta S > 0$)
- Highlights importance of whether $\Delta S > 0$?
- Not special to gravity. Interesting applications elsewhere?

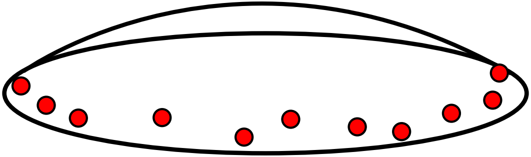
Thank You

Additional Slides

Degenerate Cases

Sometimes $\Delta M = -T\Delta S$ just reads $0 = 0$.

Seemingly non-generic: $\Delta M = 0$, $T = 0$, and $\Delta S \neq 0$



Above: No change in minimum mass at all, but ΔS changes

Degenerate Cases

A gravitational example: the Gauss-Bonnet term [1909.07983]

$$I = \frac{1}{16\pi}\int \mathrm{d}^4x\,\sqrt{-g}\left(R - F^2 + \lambda\left(R^2_{\mu\nu\rho\sigma} - 4R^2_{\mu\nu} + R^2\right)\right)$$

GB is a total derivative (so $\Delta g_{\mu\nu} = 0$), but $\Delta S \neq 0$

Extremal state still has $M = Q$, so $\Delta M = T = 0$, but $\Delta S \neq 0$

So, strictly speaking, $\Delta S > 0$ doesn't imply $\Delta M < 0$ in all cases

(Though $\Delta M < 0$ does imply $\Delta S > 0$ and similar for $\Delta M > 0$)