Shift Symmetries in Inflation

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Overview

- Goal: Understand shift-symmetric limit of inflation

$$\phi(x^\mu) \to \phi(x^\mu) + c$$

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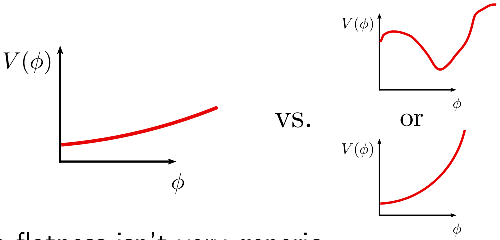
- Observational Motivation:
  - Data increasingly favors flatter potentials over steeper ones
- Theoretical Motivation:
  - Symmetry needed to naturally explain flatness of potential
  - Would generate distinctive relations between non-Gaussianities

Motivation: Puzzling Potentials

- Inflation needs to last for a while, ~ 60 e-foldings
- Typically needs shallow $V(\phi)$ over a long period



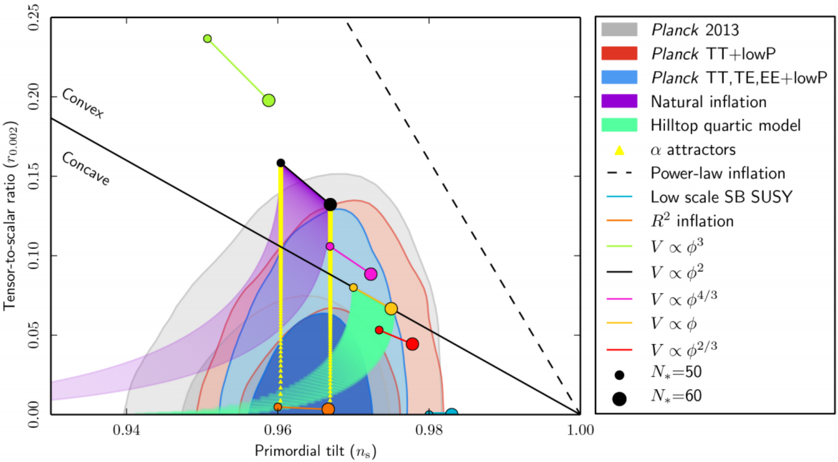
- Such flatness isn't very generic
- Worse: quantum effects compete *against* flatness (E.g. Eric's lectures)

$$V(\phi) = V_{\rm flat}(\phi) + \hbar \sum_n c_n \frac{\phi^n}{\Lambda^{n-4}} + \hbar \sum_n d_n \left(\partial\phi\right)^2 \frac{\phi^n}{\Lambda^n} + \ldots$$

- $\mathcal{O}(\hbar)$ terms are typically important, unless symmetry says otherwise
- Shift symmetry: $c_n = 0$. Approximate shift symmetry: $c_n \approx 0$

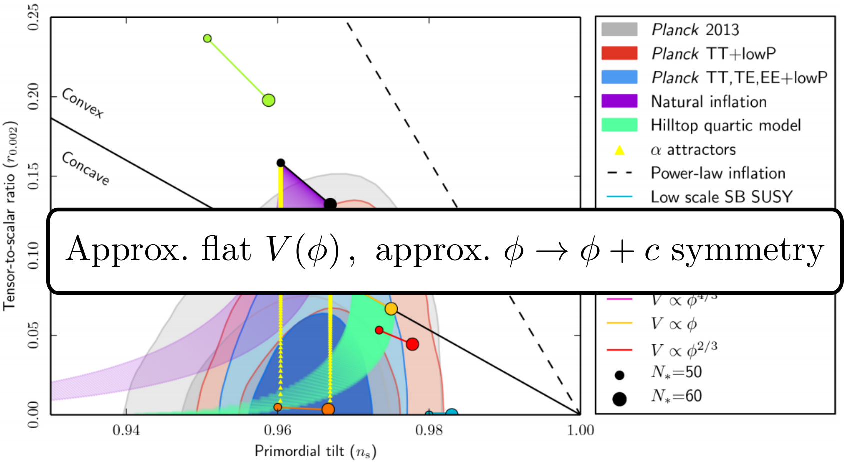
Motivation: Planck Constraints on Inflation [Planck Collaboration, 2015]

- Considering $V(\phi) \sim \phi^n$, smaller n is favored



Motivation: Planck Constraints on Inflation [Planck Collaboration, 2015]

- Considering $V(\phi) \sim \phi^n$, smaller n is favored. Pretty flat.



Approx. flat $V(\phi)$, approx. $\phi \to \phi + c$ symmetry

Symmetries $\implies$ Relations Among Non-Gaussianities

- What are the consequences of symmetries?
- E.g. Maldacena's consistency relation [Maldacena, 2002]
- Scalar perturbations of FRW: $\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2 e^{2\zeta}\mathrm{d}\mathbf{x}^2$
- Fluctuations of ϕ turn into ζ fluctuations
- Maldacena: $\zeta(\mathbf{x}) \to \zeta\left((1+\lambda)\mathbf{x}\right) + \lambda$ a symmetry

$$\implies \lim_{\mathbf{q}\to 0}\langle \zeta_{-\mathbf{q}}\zeta_{\mathbf{k}+\mathbf{q}/2}\zeta_{-\mathbf{k}+\mathbf{q}/2}\rangle' \approx \overbrace{(1-n_s)}^{\mathcal{O}(10^{-2})} P_\zeta(q)P_\zeta(k)$$

- Hailed as a smoking gun test for non-standard early universe dynamics
- A $\phi \to \phi + c$ symmetry also generates relations among NG

Ultra-Slow-Roll Inflation [Kinney, 2005]

- Find consequences of shift symmetry in a toy example:

$$S = \int \mathrm{d}^4x\,\sqrt{-g}\left(\frac{M_p^2}{2}R - \frac{1}{2}(\nabla\phi)^2 - \Lambda\right)$$

- Free scalar, no potential: $\phi \to \phi + c$ symmetry
- Near de Sitter solution with ϕ rolling along. But not slow-roll

$$\varepsilon = -\frac{\dot{H}}{H^2} \ll 1\,, \quad \eta = \frac{\dot{\varepsilon}}{\varepsilon H} \approx -6$$

- However, power spectrum is that of vanilla inflation $P_\zeta(k) = \frac{H^2}{\varepsilon M_p^2}\frac{1}{k^3}$
- Notable: Violates Maldacena's consistency relation [Namjoo et al., 2012]

$$\lim_{\mathbf{q}\to 0}\langle \zeta_{-\mathbf{q}}\zeta_{\mathbf{k}+\mathbf{q}/2}\zeta_{-\mathbf{k}+\mathbf{q}/2}\rangle' \approx \overbrace{(1-n_s)}^{6} P_\zeta(q)P_\zeta(k)$$

Shift Symmetry and Operator Product Expansions

- The shift symmetry instead predicts the right relation [GG et al, 2017]
- Want to calculate: $\lim_{\mathbf{q}\to 0}\langle\zeta_{-\mathbf{q}}\zeta_{\mathbf{k}+\mathbf{q}/2}\zeta_{-\mathbf{k}+\mathbf{q}/2}\rangle'$
- $\zeta_{\mathbf{k}+\mathbf{q}/2}\zeta_{-\mathbf{k}+\mathbf{q}/2}$ can be replaced by OPE

$$\zeta_{\mathbf{k}+\mathbf{q}/2}\zeta_{-\mathbf{k}+\mathbf{q}/2} \xrightarrow{\mathbf{q}\to 0} f(k)\zeta_{-\mathbf{q}} + g(k)\dot{\zeta}(\mathbf{q}) + \dots$$

- Maldacena predicts $f(k)$, $\phi \to \phi + c$ predicts $g(k)$
- In Slow-Roll, $g(k)$ is absent [Baumann et al., 2012]. Here, $g(k)$ dominates
- Essential difference from slow-roll: $\dot{\zeta} = 0$ in slow roll, $\dot{\zeta} > 0$ in USR

Detailed Expressions

$$\zeta_{\mathbf{k}+\mathbf{q}/2}\zeta_{-\mathbf{k}+\mathbf{q}/2} \xrightarrow{\mathbf{q}\to 0} f(k)\zeta_{-\mathbf{q}} + g(k)\dot{\zeta}(\mathbf{q})$$

- $f(k)$ depends on $P_\zeta(k)$, while new $g(k)$ term depends on $\dot{P}_\zeta(k)$
- Non-Gaussian Prediction

$$\begin{aligned}\lim_{\mathbf{q}\to 0}\langle\zeta_{-\mathbf{q}}\zeta_{\mathbf{k}+\mathbf{q}/2}\zeta_{-\mathbf{k}+\mathbf{q}/2}\rangle' &\approx (1-n_s)P_\zeta(q)P_\zeta(k) \\ &\quad + \frac{\dot{P}_\zeta(q)}{6H^2}\left[(n_s-1)\,HP_\zeta(k) + \dot{P}_\zeta(k)\right] \\ &\approx 6P_\zeta(q)P_\zeta(k)\end{aligned}$$

- Perfect match: $\phi \to \phi + c$ prediction gets the result exactly right

Conclusions

- Shift symmetric inflation observationally & theoretically interesting
- Symmetry gives definite relations between non-Gaussianities
- Explicit example: gives correct Ultra-Slow-Roll relations in a regime where celebrated Maldacena relation fails
- Future work:
  - Study more realistic models where slow roll is possible
  - Use $\phi \to \phi + c$ to constrain functions of time in EFT of Inflation (Again, see Eric's lectures)
  - Quantitatively characterize weak breaking of the shift symmetry

The End

Thank you for listening!