Spontaneous Symmetry Breaking and Massive Gravity

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Overview

- The universe is accelerating. A CC or something else?
- Modifications of gravity can change the interpretation of cosmology.
- Massive gravity is naively a straightforward alteration.
- Full of challenges, interesting physics. Highlights properties of GR.
- Ideally, mGR would be analogous to standard Higgs mechanism.
- Can we construct mGR via SSB methods?

An IR Modification of Gravity

Degravitation

- Inferred Λ assumes GR is correct at long scales.
- Long distance modifications of gravity can change the interpretation.
- Degravitation: Λ is special, sources gravity differently?
- Gravity as a high-pass filter, ex.,

$$M_{\rm pl}^2 G_{\mu\nu} = T_{\mu\nu} \longrightarrow M_{\rm pl}^2 (L^2 \Box) G_{\mu\nu} = T_{\mu\nu}$$

- Answers "Why does Λ gravitate so little?" -Arkani-Hamed et al, 0209227

dRGT (de Rham et al., 1011.1232)

- mGR a natural degravitation candidate. Finite range $\implies$ blind to Λ
- Single known non-linear theory with correct dof
- Square root matrices or vielbeins (Hinterbichler et al., 1203.5783)

$$\mathcal{L} \sim M_{\rm pl}^2 R + M_{\rm pl}^2 m^2 \epsilon_{abcd} \Big[\alpha_1 \mathbf{e}^a \wedge \mathbf{e}^b \wedge \mathbf{e}^c \wedge \mathbf{e}^d + \alpha_2 \mathbf{e}^a \wedge \mathbf{e}^b \wedge \mathbf{e}^c \wedge \mathbf{1}^d + \alpha_3 \mathbf{e}^a \wedge \mathbf{e}^b \wedge \mathbf{1}^c \wedge \mathbf{1}^d + \alpha_4 \mathbf{e}^a \wedge \mathbf{1}^b \wedge \mathbf{1}^c \wedge \mathbf{1}^d \Big] , \quad \mathbf{1}^a = \delta^a_\mu \mathrm{d}x^\mu$$

- dRGT degravitation fails for our universe
- Still, dRGT and many closely related theories are of interest.

Non-Linear Realizations

Outline

- We want analogies between Higgsed gauge theories and dRGT
- Starting point is the spontaneous symmetry breaking (SSB) pattern
- Non-linear realizations are powerful SSB method
- Technical, more commonly known for global breaking
- First review ideas, methods. Of interest for galileon like theories
- Later apply to gauge theories and gravity.

Sketch of Non-Linear Realizations: Global SSB

- Spontaneously break $G \to H$.
- Lagrangian is symmetric, $\mathcal{L}(\Phi) = \mathcal{L}(g\Phi)$, not field profile $g\Phi \neq \Phi$.
- Low energy fields, π, are different from UV ones, Φ
- π retains a "memory" of original G symmetry:

$$\pi \to a + b\pi + c\pi^2 + d\pi^3 + \dots$$

- Leads to highly restrictive, organized EFT for π.

$$\mathcal{L}_0 \sim (\partial\pi)^2 + e\pi(\partial\pi)^2 + f\pi^2(\partial\pi)^2 + \dots \ .$$

- Gross features of π physics are independent of UV

Global Internal SSB: Low Energy Degrees of Freedom

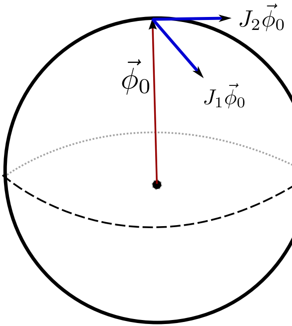
Example: $SO(3) \to SO(2)$

$$V(\vec{\phi}) = -m^2|\vec{\phi}|^2 + \lambda|\vec{\phi}|^4$$

$$|\vec{\phi}_0| = \frac{m}{2\sqrt{\lambda}}$$

$$J_3\vec{\phi}_0 = \vec{0}$$

$$J_2\vec{\phi}_0, J_1\vec{\phi}_0 \neq 0$$

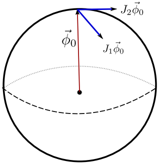


- Two massless bosons at low energies: $\vec{\phi} = \exp\left[i\pi_1 J_1 + i\pi_2 J_2\right]\vec{\phi}_0$.
- Broken symmetries $\implies$ massless bosons.

Global Internal SSB: Non-Linear Realizations

Example: $SO(3) \to SO(2)$

$$\vec{\phi} = e^{i\pi_1 J_1 + i\pi_2 J_2}\left(\vec{\phi}_0 + \hat{z}\rho\right)$$

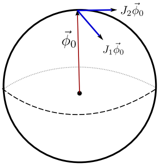


- $\vec{\phi} \to R \cdot \vec{\phi}$ *always* a symmetry. Simple for $\vec{\phi}$, sometimes messy for π.
- $J_3 : \begin{pmatrix} \pi_1 \\ \pi_2 \end{pmatrix} \to \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \pi_1 \\ \pi_2 \end{pmatrix}$
- $J_1 : \begin{pmatrix} \pi_1 \\ \pi_2 \end{pmatrix} \to \begin{pmatrix} \pi_1 + \epsilon - \frac{\epsilon}{3}\pi_2^2 \\ \pi_2 + \frac{\epsilon}{3}\pi_1\pi_2 \end{pmatrix} + \mathcal{O}(\pi^4)$
- Linear H vs. non-linear G/H symmetry behavior is a generic pattern.

Global Internal SSB: Non-Linear Realizations

Example: $SO(3) \to SO(2)$

$$J_1 : \begin{pmatrix} \pi_1 \\ \pi_2 \end{pmatrix} \to \begin{pmatrix} \pi_1 + \epsilon - \frac{\epsilon}{3}\pi_2^2 \\ \pi_2 + \frac{\epsilon}{3}\pi_1\pi_2 \end{pmatrix} + \mathcal{O}(\pi^4)$$



- Messiness is a good thing! Highly constraining.
- Kinetic term accompanied by *universal* interactions.

$$\frac{\mathcal{L}_0}{f^2} = \frac{1}{2}(\partial\pi_1)^2 + \frac{1}{2}(\partial\pi_2)^2 - \frac{1}{6}\pi_1^2(\partial\pi_2)^2 - \frac{1}{6}\pi_2^2(\partial\pi_1)^2 + \frac{1}{3}\pi_1\pi_2\partial\pi_1 \cdot \partial\pi_2 + \dots$$

- Organizes entire EFT with basic building blocks $\mathcal{L} = \sum_i c_i \mathcal{L}_i / f^{\Delta_i - 4}$.

Global Internal SSB: Non-Linear Realizations

General Features

- Goldstone symmetries only rely on SSB pattern.
- Different UV physics $\longrightarrow$ same Goldstone behavior.
- EFT construction doesn't require total understanding of UV.
- Ex. Chiral Symmetry breaking, $q = \begin{pmatrix} u \\ d \end{pmatrix}$

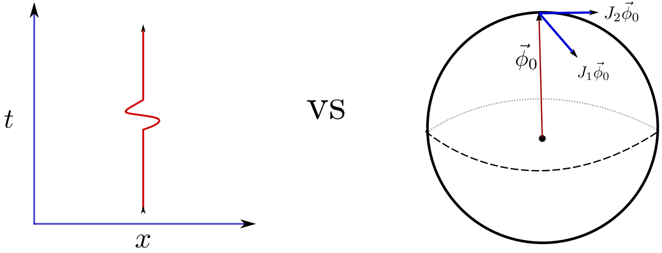
$$\mathcal{L} = i\bar{q}_L \partial\!\!\!/ q_L + i\bar{q}_R \partial\!\!\!/ q_R \implies SU(2)_L \times SU(2)_R \ .$$

- If $\langle \bar{q}q \rangle = f^3 \neq 0$, then only diagonal preserved, $\implies$ pions.
- Get pretty far without detailed understanding of breaking mechanism.
- Given $G \to H$ process is algorithmic.

Spacetime SSB vs Internal SSB

Subtleties

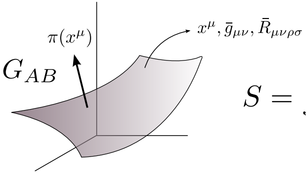
- Fewer Goldstones than broken symmetries, generally.
- Simple point particle provides intuition, (Low and Manohar, 0110285)



- Not all broken symmetries are independent.
- Combo of broken spacetime generators leaves ground state invariant.
- Ex.: phonons. Rotations and translations broken, only 3 dof.

Spacetime SSB: Non-Linear Realizations

Probe Branes



$$S = \int d^4x\sqrt{-\bar{g}}\left[c_1 + c_2 K + c_3 \bar{R} + \dots\right]$$

- Spacetime SSB has same properties as internal SSB.
- Ex.: Break 5D Minkowski with a 3-brane. Only 1 dof.
- Induced metric is $\bar{g} = \eta_{\mu\nu} + \partial_\mu\pi\partial_\nu\pi$.
- 4D Poincaré preserved, realized as usual.
- Broken 5D symmetries: $\pi \to \pi + c$ and $\pi \to b^\mu\left(x_\mu + \pi\partial_\mu\pi\right)$.
- Universal piece $\mathcal{L}_0 \sim \sqrt{1 + (\partial\pi)^2}$, EFT highly organized.

Spacetime SSB: Non-Linear Realizations

General Features

- Given, $G \to H$, EFT construction (mostly) algorithmic.
- Again, low energy description insensitive to UV.
- Ex: Conformal galileons, M_4 in AdS_5.
- Algorithm says (GG et al., 1203.3191):

$$\mathcal{L} = \frac{f^2}{2} e^{2\pi} (\partial\pi)^2 - c_1 f^4 e^{4\pi} + c_2 \left((\partial\pi)^2 \Box\pi + \frac{1}{2} (\partial\pi)^4 \right) + \dots$$

- Symmetries divorce result from specific UV picture.
- M_4 in AdS_5 is a $SO(4,2) \to SO(3,1)$ pattern.
- *Same* objects arise when breaking conformal symmetry, see a-theorem.

The Algorithm: Coset Constructions

General Procedure (CCWZ,Volkov, Ogievetsky)

- Formalizes non-linear transformations, creates invariant actions.
- Modeled after the typical internal SSB case.
- Break $G \to H$, where $\{V_I\}$'s generate H and $\{Z_a\}$'s are broken.
- Identify fields with coordinates of the coset G/H, $\tilde{g} = e^{\pi^a Z_a}$.
- $\pi \to \pi'$ transformation via $g \in G$:

$$g e^{\pi^a Z_a} = e^{\pi'^a Z_a} h(g, \pi)$$

- $h : \pi^a \to R^a{}_b \pi^b$
- $g \notin H : \pi^a \to \pi^a + c^a + c^a{}_b \pi^b + c^a{}_{bc} \pi^b \pi^c \ldots$

The Algorithm: Coset Constructions

General Procedure (CCWZ,Volkov, Ogievetsky)

- Need to build actions invariant under $\pi \to \pi'$
- Trial and error is fine, but there's a better way
- Maurer-Cartan form is the tool, $\tilde{g} = e^{\pi^a Z_a}$

$$\tilde{g}^{-1}\mathrm{d}\tilde{g} \equiv \omega_V^I V_I + \omega_Z^a Z_a = \omega_V + \omega_Z$$

- Its components transform conveniently when $\pi \to \pi'$:

$$\omega_Z \to h\omega_Z h^{-1}$$
$$\omega_V \to h\left(\omega_V + \mathrm{d}h\right)h^{-1}$$

- Lagrangians like $\mathrm{Tr}\left[\omega_Z \wedge \omega_Z \wedge \omega_Z \wedge \omega_Z\right]$ are invariant.
- Algorithm provides dof, symmetries and invariants.

The Algorithm: Coset Constructions

Inverse Higgs Constraints (CCWZ,Volkov, Ogievetsky)

- Loss of dof is only difference between internal and spacetime cases.
- Leads to (somewhat mysterious) general rule to remove fields.
- If preserved translation P, broken Z_1 have $[P, Z_1] = Z_2 + \ldots$, then set $\omega_{Z_2} \to 0$.
- Eliminates π_1 in favor of derivatives of π_2,

$$\pi_1 = \pi_1(\partial\pi_2, \pi_{i\neq 1})$$

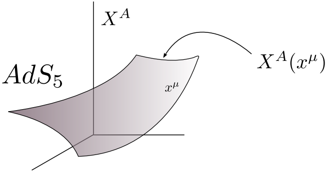
- Often equivalent to taking $\mathcal{L}(\pi_1, \pi_2, \ldots)$ and integrating out π_2.
- In any case, same number of symmetries using fewer fields.

Example: Conformal Galileons

M_4 in AdS_5 or $SO(4,2) \to SO(3,1)$

See (Bellucci et al., 0206126) or (GG et al., 1203.3191).

- One true dof.
- Dilations, SCTs broken.
- $\tilde{g} = e^{x^\mu P_\mu} e^{\pi D} e^{\xi^\mu K_\mu}$



- Single inverse Higgs constraint from $[K_\mu, P_\nu] = 2J_{\mu\nu} - 2\eta_{\mu\nu} D$
- Since $\omega_D = \mathrm{d}\pi + 2e^\pi \xi_\mu \mathrm{d}x^\mu$, we set $\xi_\mu = -\frac{1}{2} e^{-\pi} \partial_\mu \pi$
- Rest is standard: Action from wedging remaining ω's, invariant under:

$$\delta\pi = c(1 + x \cdot \partial\pi) \ , \quad \delta\pi = b \cdot x + b \cdot x \ x \cdot \partial\pi - \frac{x^2}{2} b \cdot \partial\pi$$

Gauge Theories And Non-Linear Realizations

YM Gauge Transformations

- Central utility of cosets is linear and non-linear symmetries.
- Gauge fields have this same linear vs. non-linear pattern:

$$A^a T_a \to \begin{cases} A^b \left[\delta^a_b + f^{bca}\theta^c\right] T_a & \theta^a = \text{const} \\ A^b \left[\delta^a_b + f^{bca}\theta^c(x)\right] T_a + \mathrm{d}\theta^a(x) T_a & \theta^a \neq \text{const} \end{cases}$$

- Very similar procedure to global breaking:

$$\mathcal{L}_0 \sim (\partial A)^2 + aA^2\partial A + bA^4 \to -\frac{1}{4}\mathrm{Tr}[F^2]$$

- Yang-Mills can be created from cosets (Ivanov et al., 1976)

Gauge Theories And Non-Linear Realizations

YM Gauge Transformations (Ivanov et al., 1976)

- Construction works, but it's a bit crude:

$$\theta^a(x)T_a = \theta^a T_a + \partial_\nu\theta^a x^\nu T_a + \frac{1}{2}\partial_\mu\partial_\nu\theta^a x^\mu x^\nu T_a + \dots$$
$$= \theta^a T_a + \partial_\nu\theta^a T^\nu{}_a + \frac{1}{2}\partial_\mu\partial_\nu\theta^a T^{\mu\nu}{}_a + \dots$$

- T_a's are global generators, $T^{\nu_1\dots\nu_n}{}_a$'s are local.
- Only linearly realize T_a's, $\implies G_{\text{gauge}} \to G_{\text{global}}$
- One field per "broken" generator, call it $A_{\nu_1\dots\nu_n}{}^a$.
- Inverse Higgs: All fields expressed in terms of A^a_ν and its derivatives.
- Building blocks for the action are $F^a_{\mu\nu}$, $D_\alpha F_{\mu\nu}{}^a\dots$

Gauge Theories and Non-Linear Realizations

Applications to GR

- Similar constructions lead to GR (Ogievetsky, 1973) (Ivanov, 1982)
- In YM, what happens if we break the global group? Massive phase.
- Then, we can apply similar reasoning to GR.
- Will dRGT be singled out as the broken phase?
- What other gravitational theories can we create?

Broken Gauge Theories from Cosets

Broken $U(1)$ (GG et al., 1405.5532)

- First step: Explore broken QED.
- Every generator is broken, the $T^{\nu_1 \dots \nu_n}$'s and now T, too.
- Single additional field, π, along with the $A_{\nu_1 \dots \nu_n}$'s.
- Inverse Higgs $\implies$ build using $F_{\mu\nu}$, $\partial_\alpha F_{\mu\nu}$ and now $A_\mu - \partial_\mu \pi$, too,

$$A_\mu \to A_\mu + \partial_\mu \lambda(x) \ , \quad \pi \to \pi + \lambda(x)$$

- Simply gives a massive $U(1)$ vector, in Stuckelberged form:

$$\mathcal{L} \sim -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m^2}{2} \left(A_\mu - \partial_\mu \pi \right)^2$$

- YM essentially the same.

GR from Cosets

- Construction of GR is more involved.

- It can be done in several different ways, $G \to H$.

$$\text{Diffeomorphisms} \to \text{GL} \implies \begin{cases} g_{\mu\nu} \\ \Gamma^{\alpha}_{\beta\rho} \end{cases}, \ \text{Diffs}$$

Or

$$ISO(3,1) \to SO(3,1) \implies \begin{cases} e_{\mu}{}^{a} \\ \omega_{\mu}{}^{ab} \end{cases}, \ \text{LLTs}$$

- (Kirsch, 0503024) and (Ivanov et al, 1981) respectively, for example.

- We find it useful to start with $G = \text{Diffeomorphisms} \times ISO(3,1)$.

- Work with $e_{\mu}{}^{a}$, $\omega_{\mu}{}^{ab}$ builds all of the transformations into the coset.

GR from Cosets

GR Construction (GG et al., 1405.5532)

- GR: $ISO(3,1)_{\text{gauge}} \times Diff \to SO(3,1)_{\text{global}} \times GL(d+1)$.
- Diff. generators are
$$P^{\nu_1 \ldots \nu_n}{}_{\mu} \sim x^{\nu_1} \ldots x^{\nu_n} \partial_\mu$$
- $ISO(3,1)_{\text{gauge}}$ generators are
$$P^{\nu_1 \ldots \nu_n}{}_{a} \sim x^{\nu_1} \ldots x^{\nu_n} P_a$$
$$J^{\nu_1 \ldots \nu_n}{}_{ab} \sim x^{\nu_1} \ldots x^{\nu_n} J_{ab}$$
- Only $P^{\mu}{}_{\nu}$ and J_{ab} are "preserved" as they act linearly in GR.
- $P^{\mu}{}_{\nu} : x^{\mu} \to R^{\mu}{}_{\nu} x^{\nu}$ and $J_{ab} : \omega^{ab} \to \Lambda^{a}{}_{a'} \Lambda^{b}{}_{b'} \omega^{a'b'}$

GR from Cosets

GR Construction (GG et al., 1405.5532)

- Many fields: one for each "broken" $P^{\nu_1\dots\nu_n}{}_{\mu}$, $P^{\nu_1\dots\nu_n}{}_a$ and $J^{\nu_1\dots\nu_n}{}_{ab}$.
- Can inverse Higgs down to $e_\mu{}^a$ and $\omega_\mu{}^{ab}$.
- Coset transformation laws are the usual ones:

$$J^{\nu_1\dots\nu_n}{}_{ab} : \begin{cases} e_\mu{}^a & \to \Lambda^a{}_b(x) e_\mu{}^b \\ \omega_\mu{}^{ab} & \to \Lambda^a{}_{a'}\Lambda^b{}_{b'}\omega_\mu{}^{a'b'} - \Lambda^b{}_c\partial_\mu\Lambda^{ac} \end{cases}$$

$$P^{\nu_1\dots\nu_n}{}_{\mu} : \begin{cases} e_\mu{}^a & \to \frac{\partial x^\nu}{\partial x'^\mu} e_\nu{}^a \\ \omega_\mu{}^{ab} & \to \frac{\partial x^\nu}{\partial x'^\mu}\omega_\nu{}^{ab} \end{cases}$$

- Building blocks are forms R^{ab}, e^a.
- Preserved symmetries: Contract with η_{ab} and ϵ_{abcd} $\to$ GR

Broken GR from Cosets

Breaking GR (GG et al., 1405.5532)

- GR: $ISO(3,1)_{\rm gauge} \times \mathit{Diff} \to SO(3,1)_{\rm global} \times GL(d+1)$.
- We want to preserve a smaller group, but which one?
- More symmetries preserved $\implies$ Fewer terms allowed in EFT.
- Most restrictive subgroup which allows dRGT is

$$\left(SO(3,1)_{\rm global} \times SO(3,1)_{\rm spacetime}\right)_{\rm diagonal}$$

$$e_\mu{}^a \to \Lambda^a{}_b e_\mu{}^b \ , \quad x^\mu \to \Lambda^\mu{}_\nu x^\nu$$

Broken GR from Cosets

dRGT Construction (GG et al., 1405.5532)

- dRGT comes from studying $ISO(3,1)_{\text{gauge}} \times Diff \to (SO(3,1)_{\text{global}} \times SO(3,1)_{\text{spacetime}})_{\text{diagonal}}$.
- Two new Stuckelbergs $\psi_\mu{}^\nu$, Θ^{ab} added to $e_\mu{}^a$ and $\omega_\mu{}^{ab}$
- Preserved symmetries: contract with η_{ab}, $\eta_{a\mu}$, ϵ_{abcd}, $\epsilon_{a\mu b\nu}$, etc.
- Use $\tilde{\Theta}_c{}^a \tilde{\Theta}_d{}^b R^{cd}$, $\tilde{\Theta}_b{}^a \mathbf{e}^b$ and $\Psi_\nu{}^\mu \mathrm{d}x^\nu$ for actions.
- GR terms are unchanged: $\epsilon \tilde{\Theta}\tilde{\Theta} R \wedge \tilde{\Theta}\mathbf{e} \wedge \tilde{\Theta}\mathbf{e} \sim \mathrm{d}^4x\sqrt{-g}R$.
- Anything constructed with mixed indices is new.

Broken GR from Cosets

dRGT Construction (GG et al., 1405.5532)

- Can now build dRGT mass terms,

$$\mathcal{L}_m \sim \epsilon_{ab\mu\nu}\Theta_c{}^a \mathbf{e}^c \wedge \Theta_d{}^b \mathbf{e}^d \wedge \Psi_\alpha{}^\mu \mathrm{d}x^\alpha \wedge \Psi_\beta{}^\nu \mathrm{d}x^\beta$$
$$\to \epsilon_{ab\mu\nu}\mathbf{e}^a \wedge \mathbf{e}^b \wedge \mathrm{d}x^\mu \wedge \mathrm{d}x^\nu$$

- Also, seemingly ghost free, parity violating terms:

$$\mathcal{L}'_m \sim \eta_{a\mu}\eta_{b\nu}\mathbf{e}^a \wedge \mathrm{d}x^\mu \wedge \mathbf{e}^b \wedge \mathrm{d}x^\nu$$

- Linear in lapse and shift...
- But, this is trivial on the "normal branch" where $e_\mu{}^a\eta_{a\nu}$ is symmetric.
- No such parity violating term in metric variables.
- What else can we build?

Broken GR from Cosets

dRGT Construction (GG et al., 1405.5532)

- dRGT isn't special.
- Wedging seems to produce ghost free terms.
- But any term that respects the symmetries should be included in EFT.

$$e_\mu{}^a \mathrm{d}x^\mu \to e_\mu{}^a$$

- Ex. $F(\eta^{\mu\nu} g_{\mu\nu})$ invariant under residual Lorentz symmetries.
- Consistent with quantum corrections which don't preserve dRGT.

Broken GR from Cosets

dRGT Construction (GG et al., 1405.5532)

- Cosets provide a different viewpoint for mGR.
- Helps connect mGR to familiar ideas.
- After understanding GR in this language, a slight variation leads to dRGT like theories.
- The fully non-linear theory is reproduced.
- What else might we be able to do with such constructions?

Gravitational Theories from Cosets

PM Gravity?

- On dS, a "partially massless" graviton has four dof

$$\mathcal{L} \sim \mathcal{L}^{(2)}_{\text{kinetic}} + \frac{\bar{R}}{4}\left(h^{\mu\nu}h_{\mu\nu} - \frac{1}{2}h^2\right) - \frac{\bar{R}}{12}\left(h_{\mu\nu}h^{\mu\nu} - h^2\right)$$

- Problematic longitudinal mode lost, gauge symmetry gained:

$$\delta h_{\mu\nu} = \left(\bar{\nabla}_\mu \bar{\nabla}_\nu + \frac{\bar{R}}{12}\bar{g}_{\mu\nu}\right)\alpha(x)$$

- Only the quadratic theory is known.
- Non-linear extension from cosets?

Summary

Conclusions

- Cosets efficiently deal with linear and non-linear symmetries
- Commonly applied to broken internal symmetries, but useful elsewhere
- Subtleties exist for spacetime SSB, relevant for galileon-like theories
- Gauge theories, and broken gauge theories, constructible this way
- dRGT follows from a mild breaking of GR
- Similar constructions follow for bigravity, multi-gravity

Based On

Based on work with Kurt Hinterbichler, Austin Joyce and Mark Trodden.

- GG, Kurt Hinterbichler, Austin Joyce, Mark Trodden, "*Galileons as Wess-Zumino Terms*", 1203.3191
- GG, Austin Joyce, Mark Trodden, "*Spontaneously Broken Gauge Theories and the Coset Construction*", 1405.5532
- GG, Kurt Hinterbichler, Austin Joyce, Mark Trodden, "*Einstein Gravity, Massive Gravity, Multi-Gravity and Nonlinear Realizations*", 1412.6098