Tensor NG and Partially Massless Fields

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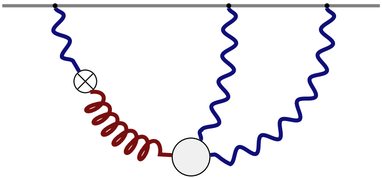
Based On [1812.07571]



UNIVERSITY OF CAMBRIDGE

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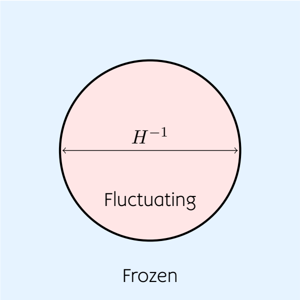
How do exotic fields affect primordial tensor non-Gaussianity $\langle\gamma^3\rangle$?

- non-Gaussianity review
- What are the vanilla expectations for tensor NG?
- What are these exotic fields?
- How can they imprint upon tensor NG?

Inflationary Perturbations

- Quantum fluctuations are imprinted on superhorizon scales



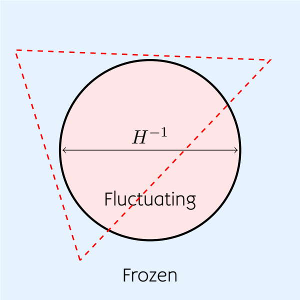
$$\longrightarrow \begin{matrix} \langle\phi\phi\rangle \\ \langle\phi\phi\phi\rangle \\ \langle\phi\phi\phi\phi\rangle \end{matrix} \longrightarrow \begin{matrix} \langle\delta T\delta T\rangle \\ \langle\delta T\delta T\delta T\rangle \\ \langle\delta T\delta T\delta T\delta T\rangle \end{matrix}$$

- These turn into correlations in the CMB/LSS

- Information: curvature size H^{-1}, departure from perfect dS, spectrum of particles, interactions, ...

Inflationary non-Gaussianity

- NG correlations needed for detailed inflationary physics



$$\longrightarrow \quad \begin{matrix} \langle \phi\phi \rangle \\ \langle \phi\phi\phi \rangle \\ \langle \phi\phi\phi\phi \rangle \end{matrix} \quad \longrightarrow \quad \begin{matrix} \langle \delta T \delta T \rangle \\ \langle \delta T \delta T \delta T \rangle \\ \langle \delta T \delta T \delta T \delta T \rangle \end{matrix}$$

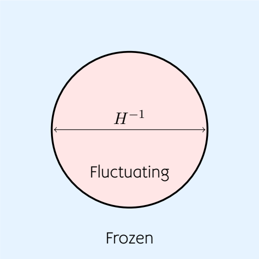
- Correlations described by prob. distribution functional

$$\mathcal{P}[\phi(\mathbf{x})] \sim \exp\left[-\frac{1}{2}\int G_2(\mathbf{x}_i)\phi(\mathbf{x}_1)\phi(\mathbf{x}_2) - \frac{1}{3!}\int G_3(\mathbf{x}_i)\phi(\mathbf{x}_1)\phi(\mathbf{x}_2)\phi(\mathbf{x}_3) + \ldots\right]$$

- NG determined by $\mathcal{L}_{\text{interactions}}$, powerful discriminator between models, since $\mathcal{L}_{\text{free}}$ essentially the same

Which Correlations Are Important?

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{M_p^2}{2}R - \frac{1}{2}(\nabla\phi)^2 - V(\phi) - \frac{1}{\Lambda^4}(\nabla\phi)^4 + \mathcal{L}_{\text{other}}(\sigma) + \dots\right)$$



Graviton, Inflaton frozen

Other fields evolve, generically

Care about **Metric** and **Inflaton** perturbations $(g_{\mu\nu}, \phi) \to (\gamma_{ij}, \zeta)$

$$\mathrm{d}s^2 = a(\tau)^2\left(-\mathrm{d}\tau^2 + e^{2\zeta}\left(\delta_{ij} + \gamma_{ij}\right)\mathrm{d}x^i\mathrm{d}x^j\right)$$

ζ, γ_{ij} determine temperature and polarization CMB fluctuations

Fields σ typically studied w/r/t influence on ζ, γ_{ij}

Recent push to understand σ's imprints upon ζ, γ_{ij}

Cosmological Collider Physics

Nima Arkani-Hamed and Juan Maldacena

Non-Gaussianity as a Particle Detector

Hayden Lee,[★] Daniel Baumann,[★,♠] and Guilherme L. Pimentel[★,♠]

Partially Massless Fields During Inflation

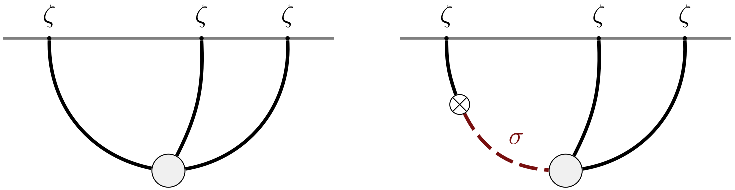
Daniel Baumann,[1] Garrett Goon[1,2] Hayden Lee,[3,4] and Guilherme L. Pimentel[1]

The Cosmological Bootstrap:

Inflationary Correlators from Symmetries and Singularities

Nima Arkani-Hamed[1], Daniel Baumann[2], Hayden Lee[3], and Guilherme L. Pimentel[2]

Sketch of previous work:



Two cases for $\langle\zeta^3\rangle$:

- Left: Single Field Inflation
- Right: Inflation + another field σ with $m \sim H$

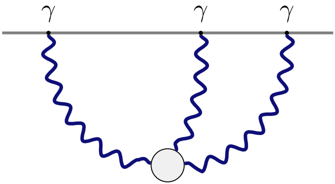
What are the signatures of the right scenario?

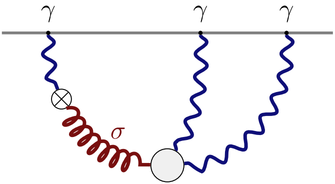
- How to distinguish σ effects from self-interactions?
- How are m, s encoded in $\langle\zeta^3\rangle$?
- How big can the induced NG be?

Answers found in [1503.08043,1607.03735]

Our Work

Our Setup:





Repeat a similar construction for tensor NG around $\approx dS$.

Why this is interesting:

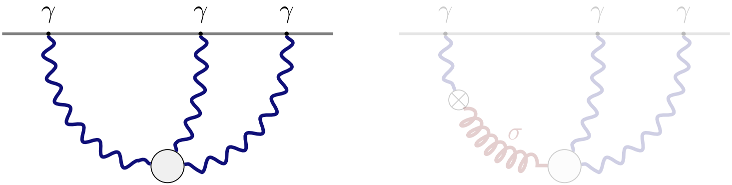
- Left scenario extremely constrained in perfect dS
- Light spinning dS fields have novel properties, no flat analogue

Recent work on probing $\langle\gamma^3\rangle$ with LISA and pulsar timing arrays

[Bartolo et al., 1806.02819] [Tsuneto et al., 1812.10615]

[Dimastrogiovanni et al., 1810.08866]

Vanilla scenario for $\langle\gamma^3\rangle$:



$\langle\gamma^3\rangle$ extremely constrained when $S = S[g_{\mu\nu}, \phi]$ and $\approx dS$

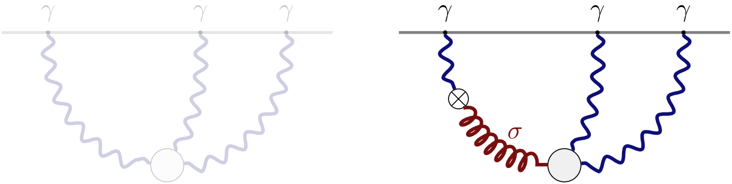
$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{M_p^2}{2}R + \mathcal{L}(\phi, g_{\mu\nu}, R) + R^2 + \frac{1}{\Lambda^{2n}}R^{2+n}\ldots\right)$$

Only two shapes for $\langle\gamma^3\rangle$ [Maldacena et al.,1104.2846]

Everything above is redundant with Einstein-Hilbert & $W^3_{\mu\nu\rho\sigma}$

Restricted form of GR shapes is a discriminator

Cosmological Collider Physics for $\langle\gamma^3\rangle$:



Include a spin-s field σ of mass m.

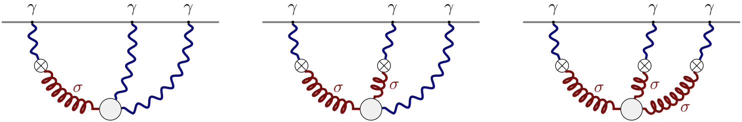
$$S \sim \int \mathrm{d}^4x\sqrt{-g}\left(\ldots + (\partial\sigma)^2 - m^2\sigma^2 + \sigma\gamma + \sigma\gamma^2 + \ldots\right)$$

Some comments:

- For the mixing to happen, $s \geq 2$
- Expect strongest effects from $m^2 \approx 2H^2$ (explained later)

Results

Results for Shapes:



There are many types of vertices one may add.

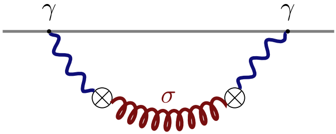
End result: middle diagrams generate **five** distinct shapes

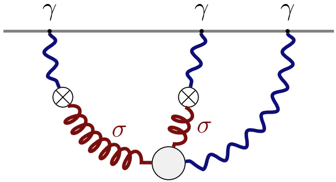
σ causes some NG shapes to be non-zero

doesn't just change $\langle\gamma^3\rangle_{\mathrm{GR}} \to \langle\gamma^3\rangle_{\mathrm{GR}} \times (1+\epsilon)$

New shapes distinguish this from the vanilla scenario

Results for Sizes:



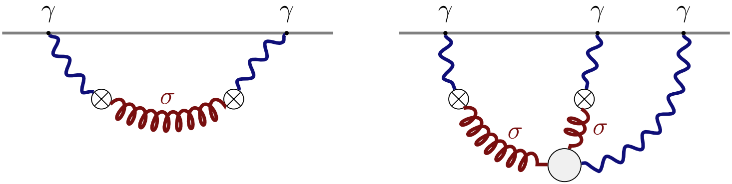


Size of $\langle\gamma^3\rangle$ depends on mixing $\otimes$ and vertex $\bigcirc$

$$S \sim \int \mathrm{d}^4x\sqrt{-g}\left(\ldots + \lambda\sigma\gamma + \frac{\partial^n}{\Lambda^{n-4}}\sigma^2\gamma + \ldots\right)$$

- λ should be small to not affect tensor power spectrum P_γ
- Λ should be large to avoid strong coupling ($\Lambda \gg H$)

Results for Sizes:



Compare to the Einstein-Hilbert result

In most optimistic regimes, the NG can be much larger:

$$\frac{\langle\gamma^3\rangle'_{\rm GR}}{\langle\gamma^2\rangle'^2} \sim 1\,, \quad \frac{\langle\gamma^3\rangle'_{\sigma}}{\langle\gamma^2\rangle'^2} \lesssim \frac{M_p}{H}\,, \quad \frac{M_p}{H} \gtrsim 10^5$$ [PLANCK,1807.06211]

(The $W^3_{\mu\nu\rho\sigma}$ operator can also produce $\frac{\langle\gamma^3\rangle'}{\langle\gamma^2\rangle'^2} \lesssim \frac{M_p}{H}$)

Tensor power spectrum $\langle\gamma^2\rangle$ negligibly affected

Main Messages

- Additional fields can generate new shapes for $\langle \gamma^3 \rangle$
- Tensor NG can be much larger than vanilla scenario
- Leaves $\langle \gamma^2 \rangle$, $\langle \zeta^2 \rangle$ and $\langle \zeta^3 \rangle$ unaffected

Details

What to Compute: Equal Time Correlators

Cosmological correlators are evaluated at equal times

More similar to Quantum Mechanics than S-matrix

$$\Psi[q(t)] \longrightarrow \langle \hat{q}(t)^n \rangle = \int \mathrm{d}q \, |\Psi[q(t)]|^2 q^n$$

Various ways to compute "in-in" correlators. We use $\Psi[\bar{\varphi}(\mathbf{k},t)]$:

$$\Psi[\bar{\varphi}(\mathbf{k},t)] \longrightarrow \langle \hat{\varphi}(\mathbf{k}_1,t)\ldots\hat{\varphi}(\mathbf{k}_n,t)\rangle = \int \mathcal{D}\bar{\varphi}\, |\Psi[\bar{\varphi}(\mathbf{k},t)|^2 \bar{\varphi}(\mathbf{k}_1,t)\ldots\bar{\varphi}(\mathbf{k}_n,t) \ .$$

"Wavefunction of the universe"

Calculating and Using Ψ

Ψ is calculated semiclassically

$$\Psi[\bar{\varphi}(\mathbf{k}, t_\star)] = \int_{\text{vac.}}^{\bar{\varphi}} \mathcal{D}\varphi \, e^{iS[\varphi]} \approx \exp\left(iS_{\text{cl.}}[\varphi_{\text{cl}}[\bar{\varphi}]]\right)$$

φ_{cl} is the classical solution equal to $\bar{\varphi}$ at $t = t_\star$

$$\Psi[\bar{\varphi}(\mathbf{k}, t_\star)] = \exp\left[-\frac{1}{2}\int \bar{\varphi}^2 \langle \mathcal{O}^2\rangle - \frac{1}{3!}\int \bar{\varphi}^3 \langle \mathcal{O}^3\rangle - \ldots\right]$$

Equal-time correlators built from $\langle \mathcal{O}^n\rangle$'s

$$\langle \hat{\varphi}(\mathbf{k}_1, t_\star)\hat{\varphi}(\mathbf{k}_2, t_\star)\rangle \sim \int \mathcal{D}\bar{\varphi}\, |\Psi|^2 \bar{\varphi}^2 \sim \frac{1}{\text{Re}\,\langle \mathcal{O}^2\rangle}$$

$$\langle \hat{\varphi}(\mathbf{k}_1, t_\star)\hat{\varphi}(\mathbf{k}_2, t_\star)\hat{\varphi}(\mathbf{k}_3, t_\star)\rangle \sim \int \mathcal{D}\bar{\varphi}\, |\Psi|^2 \bar{\varphi}^3 \sim \frac{\text{Re}\,\langle \mathcal{O}^3\rangle}{\text{Re}\,\langle \mathcal{O}^2\rangle^3}$$

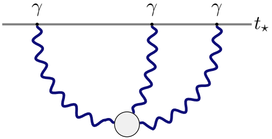
Diagrams and On-Shell Interactions

Our focus is on the cubic coefficients in Ψ

These are cubic interactions evaluated on-shell

Example: cubic γ coefficient

$$\ln\Psi \supset -\frac{1}{3!}\int \bar{\gamma}^3\langle T^3\rangle$$



Above Diagram:

- Take cubic action $S_3[\gamma]$
- Set $\gamma \longrightarrow \gamma_{\rm cl}$, γ obeys EOM and $\gamma_{\rm cl}(t_\star) = \bar{\gamma}$
- Integrate over all space, and time up to $t = t_\star$
- $S_3[\gamma_{\rm cl}] = -\frac{1}{3!}\int \bar{\gamma}^3\langle T^3\rangle$

Finding on-shell cubic interactions is bulk of work

An Aside: Ψ and AdS/CFT

Ψ is the central object in the Holographic dictionary

$$\Psi_{dS}[\bar{\varphi}] \longleftrightarrow Z_{AdS}[\bar{\varphi}]$$

Notation and form are the same:

$$\Psi[\bar{\varphi}(\mathbf{k}, t_\star)] = \exp\left[-\frac{1}{2}\int \bar{\varphi}^2 \langle \mathcal{O}^2 \rangle - \frac{1}{3!}\int \bar{\varphi}^3 \langle \mathcal{O}^3 \rangle - \ldots\right]$$

But the use is different

$$\text{AdS/CFT} \quad \frac{\delta^3 Z}{\delta \bar{\varphi}^3} \sim \langle \mathcal{O}^3 \rangle$$

$$\text{Cosmology} \quad \int \mathcal{D}\bar{\varphi}\, |\Psi|^2 \bar{\varphi}^3 \sim \frac{1}{\mathrm{Re}\,\langle \mathcal{O}^2 \rangle^3}\, \mathrm{Re}\,\langle \mathcal{O}^3 \rangle$$

Many AdS/CFT techniques apply for non-Gaussianities

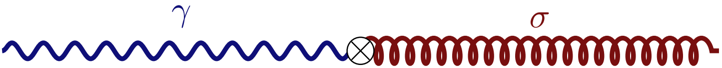
Steps of the Calculation

- Find on-shell cubic interactions between γ and σ
- Calculate coefficients $\langle\Sigma^3\rangle$, $\langle T\Sigma^2\rangle$, $\langle \Sigma T^2\rangle$
- Construct $\langle\gamma^3\rangle$ with these building blocks (and $\langle T\Sigma\rangle$)

$$\Psi[\bar{\gamma},\bar{\sigma}] \sim \exp\left[-\frac{1}{2}\gamma^2\langle T^2\rangle - \frac{1}{2}\sigma^2\langle\Sigma^2\rangle - \gamma\sigma\langle T\Sigma\rangle - \frac{1}{2}\gamma\sigma^2\langle T\Sigma^2\rangle + \dots\right]$$

What is an interesting choice of σ?

- σ must have spin ≥ 2
- Lighter σs expected to give bigger signal
- Lightest non-massless σ on dS very non-trivial. Our focus



Take  to be a Spin-2 Field

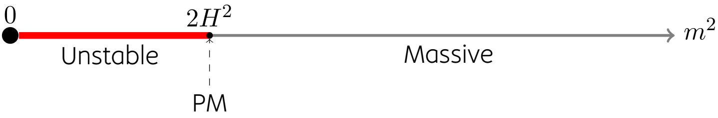
"Higuchi Bound": Spin-2 fields must have $m^2 \geq 2H^2$

$$S = \int \mathrm{d}^4x\sqrt{-g}\Big(-\frac{1}{2}\nabla^\alpha\sigma^{\mu\nu}\nabla_\alpha\sigma_{\mu\nu} + \frac{1}{2}\nabla^\alpha\sigma\nabla_\alpha\sigma + \dots$$
$$- \left(H^2 + \frac{m^2}{2}\right)\sigma_{\mu\nu}\sigma^{\mu\nu} - \frac{1}{2}\left(H^2 - m^2\right)\sigma^2\Big)$$

Otherwise, some components acquire wrong-sign kinetic terms

Suggests looking at $m^2 = 2H^2$. Very special point!

Partially Massless Fields



Spin-2 with $m^2 = 2H^2$ has **4** DOF, $m^2 > 2H^2$ has **5**

σ has a **gauge symmetry** at PM point

$$\sigma_{\mu\nu} \to \sigma_{\mu\nu} + \left(\nabla_\mu \nabla_\nu + H^2 \bar{g}_{\mu\nu}\right) \alpha(x^\mu)$$

Special to dS, no flat analogue. A smoking gun?

Technical advantage: PM mode functions nice, $\sigma \sim \tau e^{ik\tau}$

Similar stories exist for higher spin fields

Partially Massless Fields: Some Background

The dream: a non-linear PM theory would make the smallness of Λ technically natural. Λ tied to a gauge symmetry.

$$\sigma_{\mu\nu} \to \sigma_{\mu\nu} + \left(\nabla_\mu \nabla_\nu + H^2 \bar{g}_{\mu\nu}\right) \alpha(x^\mu)$$

The difficulties: constructing interactions challenging

- No-Go: isolated spin-2 PM field can't self-interact consistently, quartic order obstruction [de Rham et al, 1302.0025]
- Require additional fields for consistency. Completion $\sim$Vasiliev?
- Unclear how to extend away from dS

Constructing consistent $\gamma - \sigma$ interactions large portion of project

Gauge-Invariant Interactions

Main Point: going from linear to non-linear theory is hard!

Consider building GR from similar starting point

$$\mathcal{L} \sim (\partial h)^2 \ , \quad h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

Adding some $\sim \partial^2 h^3$ terms makes system non gauge-invariant

Need to simultaneously also alter the gauge symmetry

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu + \mathcal{O}\left(\partial h \xi\right) \ , \quad \mathcal{L} \sim (\partial h)^2 + \mathcal{O}\left(\partial^2 h^3\right)$$

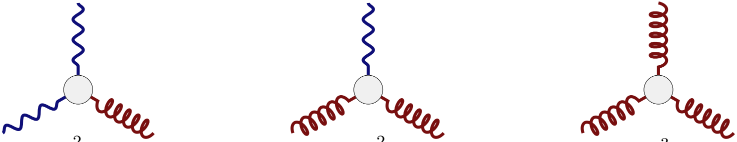
Eventually, everything is repackaged into GR

$$g_{\mu\nu} \to \frac{\partial x'^\alpha}{\partial x^\mu} \frac{\partial x'^\beta}{\partial x^\nu} g_{\alpha\beta} \ , \quad S \sim \int \mathrm{d}^4 x \sqrt{-g}\, R$$

But pretty difficult to get there without knowing answer!

No guarantee system will close, in general

Some Simplifications



Find interactions consistent with **both** σ/γ gauge symmetries

A few simplifications:

- Only need cubic interactions, not whole non-linear theory
- $\langle\gamma^3\rangle$ computations only require **on-shell** interactions

E.g., for massless spin-2 $(\gamma_{ij}/h_{\mu\nu})$ we can use

$$\Box h_{\mu\nu} = 2H^2 h_{\mu\nu}\ , \quad \nabla^\mu h_{\mu\nu} = 0\ , \quad h^\mu{}_\mu = 0$$

Interaction Simplifications

Example: GR has **33** cubic terms, off-shell:

$$S = \frac{M_p^2}{2}\int \mathrm{d}^4x\sqrt{-g}\,\left(R-6H^2\right) \supset$$

$$-2\,H^2\,\mathrm{hh}^{1}{}_{a}{}^{c}\,\mathrm{hh}^{1ab}\,\mathrm{hh}^{1}{}_{bc} + \frac{3}{2}H^2\,\mathrm{hh}^{1a}{}_{a}\,\mathrm{hh}^{1}{}_{bc}\,\mathrm{hh}^{1bc} - \frac{1}{4}H^2\,\mathrm{hh}^{1a}{}_{a}\,\mathrm{hh}^{1b}{}_{b}\,\mathrm{hh}^{1c}{}_{c} - \frac{3}{4}\mathrm{hh}^{1ab}\,\nabla_a\mathrm{hh}^{1cd}\,\nabla_b\mathrm{hh}^{1}{}_{cd} + \frac{1}{4}\mathrm{hh}^{1ab}\,\nabla_a\mathrm{hh}^{1c}{}_{c}\,\nabla_b\mathrm{hh}^{1d}{}_{d} - \mathrm{hh}^{1ab}\,\nabla_b\mathrm{hh}^{1d}{}_{d}\,\nabla_c\mathrm{hh}^{1}{}_{a}{}^{c} - \mathrm{hh}^{1ab}\,\nabla_b\mathrm{hh}^{1}{}_{a}{}^{c}\,\nabla_c\mathrm{hh}^{1d}{}_{d} -$$

$$\mathrm{hh}^{1}{}_{a}{}^{c}\,\mathrm{hh}^{1ab}\,\nabla_c\nabla_b\mathrm{hh}^{1d}{}_{d} + \frac{1}{2}\mathrm{hh}^{1a}{}_{a}\,\mathrm{hh}^{1bc}\,\nabla_c\nabla_b\mathrm{hh}^{1d}{}_{d} + \mathrm{hh}^{1}{}_{a}{}^{c}\,\mathrm{hh}^{1ab}\,\nabla_c\nabla_d\mathrm{hh}^{1}{}_{b}{}^{d} - \frac{1}{2}\mathrm{hh}^{1a}{}_{a}\,\mathrm{hh}^{1bc}\,\nabla_c\nabla_d\mathrm{hh}^{1}{}_{b}{}^{d} + \frac{1}{2}\mathrm{hh}^{1ab}\,\nabla_c\mathrm{hh}^{1d}{}_{d}\,\nabla^c\mathrm{hh}^{1}{}_{ab} - \frac{1}{8}\mathrm{hh}^{1a}{}_{a}\,\nabla_c\mathrm{hh}^{1d}{}_{d}\,\nabla^c\mathrm{hh}^{1b}{}_{b} + \mathrm{hh}^{1ab}\,\nabla_c\mathrm{hh}^{1}{}_{a}{}^{c}\,\nabla_d\mathrm{hh}^{1}{}_{b}{}^{d} +$$

$$2\,\mathrm{hh}^{1ab}\,\nabla_b\mathrm{hh}^{1}{}_{a}{}^{c}\,\nabla_d\mathrm{hh}^{1}{}_{c}{}^{d} - \frac{1}{2}\mathrm{hh}^{1a}{}_{a}\,\nabla_b\mathrm{hh}^{1bc}\,\nabla_d\mathrm{hh}^{1}{}_{c}{}^{d} - \mathrm{hh}^{1ab}\,\nabla^c\mathrm{hh}^{1}{}_{ab}\,\nabla_d\mathrm{hh}^{1}{}_{c}{}^{d} + \frac{1}{2}\mathrm{hh}^{1a}{}_{a}\,\nabla^c\mathrm{hh}^{1b}{}_{b}\,\nabla_d\mathrm{hh}^{1}{}_{c}{}^{d} + \mathrm{hh}^{1ab}\,\mathrm{hh}^{1cd}\,\nabla_d\nabla_b\mathrm{hh}^{1}{}_{ac} - \mathrm{hh}^{1ab}\,\mathrm{hh}^{1cd}\,\nabla_d\nabla_c\mathrm{hh}^{1}{}_{ab} + \mathrm{hh}^{1}{}_{a}{}^{c}\,\mathrm{hh}^{1ab}\,\nabla_d\nabla_c\mathrm{hh}^{1}{}_{b}{}^{d} -$$

$$\frac{1}{2}\mathrm{hh}^{1a}{}_{a}\,\mathrm{hh}^{1bc}\,\nabla_d\nabla_c\mathrm{hh}^{1}{}_{b}{}^{d} - \frac{1}{4}\mathrm{hh}^{1}{}_{ab}\,\mathrm{hh}^{1ab}\,\nabla_d\nabla_c\mathrm{hh}^{1cd} + \frac{1}{8}\mathrm{hh}^{1a}{}_{a}\,\mathrm{hh}^{1b}{}_{b}\,\nabla_d\nabla_c\mathrm{hh}^{1cd} - \mathrm{hh}^{1}{}_{a}{}^{c}\,\mathrm{hh}^{1ab}\,\nabla_d\nabla^d\mathrm{hh}^{1}{}_{bc} + \frac{1}{2}\mathrm{hh}^{1a}{}_{a}\,\mathrm{hh}^{1bc}\,\nabla_d\nabla^d\mathrm{hh}^{1}{}_{bc} + \frac{1}{4}\mathrm{hh}^{1}{}_{ab}\,\mathrm{hh}^{1ab}\,\nabla_d\nabla^d\mathrm{hh}^{1c}{}_{c} -$$

$$\frac{1}{8}\mathrm{hh}^{1a}{}_{a}\,\mathrm{hh}^{1b}{}_{b}\,\nabla_d\nabla^d\mathrm{hh}^{1c}{}_{c} + \mathrm{hh}^{1ab}\,\nabla_b\mathrm{hh}^{1}{}_{cd}\,\nabla^d\mathrm{hh}^{1}{}_{a}{}^{c} + \frac{1}{2}\mathrm{hh}^{1ab}\,\nabla_c\mathrm{hh}^{1}{}_{bd}\,\nabla^d\mathrm{hh}^{1}{}_{a}{}^{c} - \frac{3}{2}\mathrm{hh}^{1ab}\,\nabla_d\mathrm{hh}^{1}{}_{bc}\,\nabla^d\mathrm{hh}^{1}{}_{a}{}^{c} - \frac{1}{4}\mathrm{hh}^{1a}{}_{a}\,\nabla_c\mathrm{hh}^{1}{}_{bd}\,\nabla^d\mathrm{hh}^{1bc} + \frac{3}{8}\mathrm{hh}^{1a}{}_{a}\,\nabla_d\mathrm{hh}^{1}{}_{bc}\,\nabla^d\mathrm{hh}^{1bc}$$

But only **3** after imposing $h^\mu{}_\mu = \nabla^\mu h_{\mu\nu} = 0$ and $\Box h_{\mu\nu} = 2H^2$

$$\frac{3}{4}H^2\,\mathrm{Mp}^2\,\mathrm{hh}^{1}{}_{a}{}^{c}\,\mathrm{hh}^{1ab}\,\mathrm{hh}^{1}{}_{bc} + \frac{1}{8}\mathrm{Mp}^2\,\mathrm{hh}^{1ab}\,\nabla_a\mathrm{hh}^{1cd}\,\nabla_b\mathrm{hh}^{1}{}_{cd} + \frac{1}{4}\mathrm{Mp}^2\,\mathrm{hh}^{1ab}\,\nabla_c\mathrm{hh}^{1}{}_{bd}\,\nabla^d\mathrm{hh}^{1}{}_{a}{}^{c}$$

Obviously better if we can work on-shell everywhere

On-Shell Gauge Invariance

Gauge invariance conditions simplify on-shell

$$h_{\mu\nu} \to h_{\mu\nu} + \delta_0 h_{\mu\nu} + \delta_1 h_{\mu\nu} + \dots \,, \quad S[h] \;= S_2[h] + S_3[h] + \dots$$

Gauge invariance means

$$0 = \int \delta_0 h_{\mu\nu} \frac{\delta S_2}{\delta h_{\mu\nu}} \,, \quad 0 = \int \delta_0 h_{\mu\nu} \frac{\delta S_3}{\delta h_{\mu\nu}} + \delta_1 h_{\mu\nu} \frac{\delta S_2}{\delta h_{\mu\nu}} \,, \quad \dots$$

But $\frac{\delta S_2}{\delta h_{\mu\nu}} = 0$ on-shell, so **only $\delta_0 h_{\mu\nu}$ is needed**

$$\int \delta_0 h_{\mu\nu} \frac{\delta S_3}{\delta h_{\mu\nu}} \Big|_{h=\text{linear solution}} \cong 0$$

Equality up to total derivatives and on-shell conditions

[Variational Derivative, Going On-Shell] $\neq 0$

Working on-shell isn't entirely painless. Finding TD's trickier

E.g., take a massless scalar $\Box\varphi = 0$. Off-shell we have:

$$\mathcal{L}_{\rm TD} = \nabla^\mu \left(\varphi^2 \nabla_\mu \varphi\right) = \varphi^2 \Box\varphi + 2\varphi(\nabla\varphi)^2 \implies \frac{\delta S_{\rm TD}}{\delta\varphi} = 0$$

But if we go on-shell and impose $\Box\varphi = 0$, then:

$$\mathcal{L}_{\rm TD}^{\rm on-shell} = 2\varphi(\nabla\varphi)^2 \implies \frac{\delta S_{\rm TD}^{\rm on-shell}}{\delta\varphi} \neq 0$$

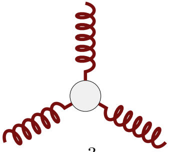
So, the problem is really to solve

$$\int \delta_0 h_{\mu\nu} \frac{\delta S_3}{\delta h_{\mu\nu}}\Big|_{h=\text{linear solution}} \cong 0 + \text{secret total derivatives}$$

We can ennumerate the relevant secret TD's

Strategy: Build Basis of Interactions

Basis of independent on-shell cubic terms is relatively small.



$$\mathcal{L}_3[\sigma] = a_1\sigma^{\mu}{}_{\nu}\sigma^{\nu}{}_{\sigma}\sigma^{\sigma}{}_{\mu} + a_2\sigma^{\rho\sigma}\nabla_\rho\sigma^{\mu\nu}\nabla_\sigma\sigma_{\mu\nu} + a_3\sigma^{\mu\rho}\nabla_\nu\sigma^{\sigma}{}_{\rho}\nabla_\sigma\sigma^{\nu}{}_{\mu} + a_4\nabla_\mu\sigma^{\kappa\rho}\nabla_\nu\sigma^{\sigma}{}_{\kappa}\nabla_{(\rho}\nabla_{\sigma)}\sigma^{\mu\nu} + a_5\nabla_\mu\nabla_\nu\sigma^{\lambda\kappa}\nabla_\rho\nabla_\sigma\sigma^{\mu\nu}\nabla_\lambda\nabla_\kappa\sigma^{\rho\sigma}$$

Imposing PM gauge invariance leads to conditions on a_i's

$$a_1 = -16a_5H^6 - 10a_4H^4 + 3a_3H^2,$$
$$a_2 = -6a_5H^4 - 3a_3H^2 + \frac{1}{2}a_3,$$

Similar bases for $\gamma\sigma^2$ and $\gamma^2\sigma$ interactions

We (mostly) reproduce interactions previously derived using embedding space [1203.6578]

Computing $\langle\gamma^3\rangle$: Mixing Required

Cubic coefficients follow easily from on-shell actions

σ and γ need to linearly mix, for σ to affect $\langle\gamma^3\rangle$, $\langle T\Sigma\rangle \neq 0$

$$\Psi[\bar{\gamma},\bar{\sigma}] \sim \exp\left[-\frac{1}{2}\gamma^2\langle T^2\rangle - \frac{1}{2}\sigma^2\langle\Sigma^2\rangle - \gamma\sigma\langle T\Sigma\rangle - \frac{1}{2}\gamma^2\sigma\langle T^2\Sigma\rangle + \ldots\right]$$

$$\langle\gamma^3\rangle \sim \frac{1}{\mathrm{Re}\,\langle T^2\rangle^3}\frac{\mathrm{Re}\,\langle T\Sigma\rangle}{\mathrm{Re}\,\langle\Sigma^2\rangle}\,\mathrm{Re}\,\langle T^2\Sigma\rangle$$

Impossible in perfect dS. Consistent w/ [Maldacena et al.,1104.2846]

But inflation isn't perfect dS. Minimally violent $\langle T\Sigma\rangle$:

$$\langle T_{\mathbf{k}}\Sigma_{-\mathbf{k}}\rangle' \propto \varepsilon k^2$$

Preserves max. possible dS symmetries while allowing mixing

Result: Using $\langle T\Sigma\rangle$, five shapes for $\langle\gamma^3\rangle$ are found

(Explicit expressions aren't very illuminating)

Ambiguities: Integration by Parts

Something Surprising: Integrations by Parts matter.

E.g., massless scalar φ on dS

$$S_1[\varphi] = \int \mathrm{d}^4x\sqrt{-g}\left(-\frac{1}{2}(\nabla\varphi)^2 + \frac{\lambda}{2}\varphi^2\Box\varphi\right)$$

$$S_2[\varphi] = \int \mathrm{d}^4x\sqrt{-g}\left(-\frac{1}{2}(\nabla\varphi)^2 - \lambda\varphi(\nabla\varphi)^2\right)$$

$$= S_1[\varphi] + \int_{\tau=\tau_\star} \mathrm{d}^3x\sqrt{h}\,\frac{\lambda}{2}n^\mu\varphi^2\nabla_\mu\varphi$$

Calculate $\langle\varphi^3\rangle$ using both $S_1[\varphi]$ and $S_2[\varphi]$

$$\langle\varphi_{\mathbf{k}_1}\varphi_{\mathbf{k}_2}\varphi_{\mathbf{k}_3}\rangle'_{S_1} = 0$$

$$\langle\varphi_{\mathbf{k}_1}\varphi_{\mathbf{k}_2}\varphi_{\mathbf{k}_3}\rangle'_{S_2} = \lambda\sum_{i\neq j}P_\varphi(k_i)P_\varphi(k_j)$$

Differ by Local non-Gaussianity

Ambiguities: Integration by Parts

Similar results for σ and γ

Very general. Another example:

$$\mathcal{L}_{\rm GB} = R^2 - 4R_{\mu\nu}^2 + R_{\mu\nu\rho\sigma}^2$$

$\mathcal{L}_{\rm GB}$ is a TD in 4D, but generates NG unless boundary term added

Boundary terms important. How to choose them in general?

Variational principle can select one, sometimes

$$\text{e.g., GHY} \quad S_{\rm GR} \sim \int_{\mathcal{M}} \sqrt{-g}R \pm \int_{\partial\mathcal{M}} \sqrt{h}K$$

But not always:

$$S_{W^3} \sim \int_{\mathcal{M}} \sqrt{-g}W_{\mu\nu\rho\sigma}^3 + \int_{\partial\mathcal{M}} \sqrt{h} \times (\quad ? \quad)$$

Ambiguities: Integration by Parts

Some good news: ambiguous parts not entirely arbitrary

Integrations by parts $\longleftrightarrow$ Local Field Redefinitions

$$\mathcal{L}[\varphi] \to \mathcal{L}[\varphi] + \nabla_\mu J^\mu[\varphi] \iff \varphi(\mathbf{x}) \to \varphi(\mathbf{x}) + \lambda\varphi(\mathbf{x})^2$$

E.g., take massless φ, add all $\mathcal{O}(\varphi^3)$ boundary terms up to $\mathcal{O}(\nabla^5)$

$$\langle \varphi_{\mathbf{k}_1}\varphi_{\mathbf{k}_2}\varphi_{\mathbf{k}_3}\rangle'_{\mathcal{L}+\text{total derivatives}} = \langle \varphi_{\mathbf{k}_1}\varphi_{\mathbf{k}_2}\varphi_{\mathbf{k}_3}\rangle'_{\mathcal{L}} + \lambda \sum_{i\neq j} P_\varphi(k_i)P_\varphi(k_j)$$

Known in holography?

Conclusions

Conclusions

Results

- Exotic dS fields can create new, large $\langle\gamma^3\rangle$ shapes, while leaving $\langle\gamma^2\rangle$, $\langle\zeta^2\rangle$ and $\langle\zeta^3\rangle$ unaffected
- Biggest challenge is consistently coupling σ to γ on dS
- Need to move slightly away from dS to imprint on $\langle\gamma^3\rangle$

Future Work

- Consistent couplings at higher order/FRW? New ingredients?
- Improved understanding of IBP/Boundary Term subtleties
- Shapes different from GR. Quantifying how different? Also different from other mechanisms?

Thank you!