Superluminality, Black Holes and EFT

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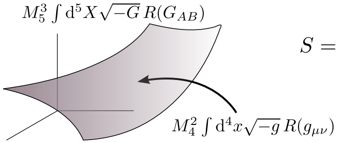
Outline

- Goal: Understand superluminality within EFT framework.
- Any sensible theory should avoid superluminality, $c_s^{\rm group} > 1$.
- However, interesting superluminal Effective Field Theories exist.
- E.g. speculative modified gravity models, but also QED.
- A natural test distinguishes the superluminality in our two examples.
- Along the way, we will see some neat black hole physics

DGP, Massive GR and Galileons

- Late time acceleration very exciting (and confusing).
- Generated an industry of GR modifications.
- Two such theories closely connected: DGP and massive GR (dRGT).

DGP



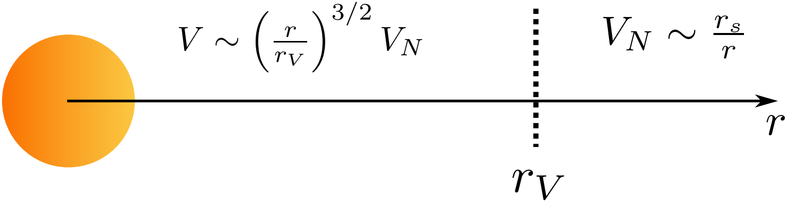
massive GR

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[M_{pl}^2 R + V(h_{\mu\nu}, \eta_{\mu\nu})\right]$$

- New light dof $\pi(x)$ appears, needs to be screened in some way.
- Same $\pi(x)$, same screening in both theories.
- Screening mechanism leads to superluminalities.

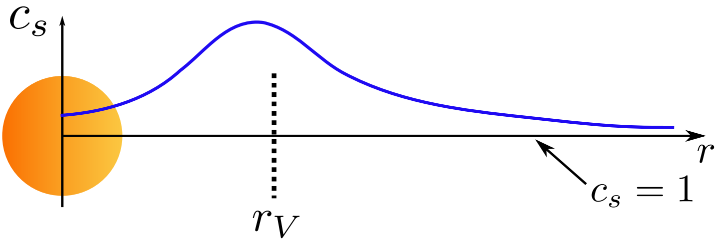
Cubic Galileon: The Good News

- Cubic Galileon: $\mathcal{L} = -\frac{1}{2}(\partial\pi)^2 - \frac{1}{\Lambda^3}(\partial\pi)^2\Box\pi + \frac{\pi}{M_{pl}}T^{\mu}{}_{\mu}$.
- $\pi(x) \to \pi(x) + c + b_\mu x^\mu$ (Nicolis, Rattazzi, Trincherini 2008)



- Non-linearities suppress potential V at $r < r_V$. "Vainshtein."
- Fifth force shuts off below $r_V = \Lambda^{-1}\left(M/M_{pl}\right)^{1/3}$.
- For us, $\Lambda^{-1} \sim 10^3$km, $r_V^{\odot} \sim 10^{15}$km ($\gg r_{\rm Solar\ System} \sim 10^9$km).

Cubic Galileon: The Bad News



- The non-linearities needed for screening also induce superluminality.
- Radially moving perturbations have $c_s > 1$.
- Travel along $\tilde{g}_{\mu\nu} = \eta_{\mu\nu} - \frac{4}{\Lambda^4}\eta_{\mu\nu}\partial^2\bar{\pi} + \frac{4}{\Lambda^3}\partial_\mu\partial_\nu\bar{\pi}$. Lightcone widens.

Comparisons?

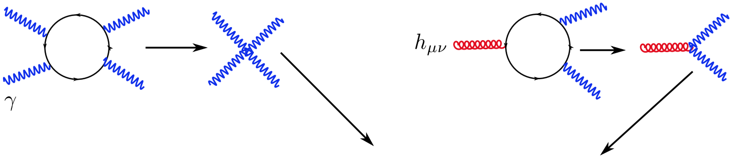
- How bad is this?
- Each theory is highly speculative.
- Are there any theories we trust with similar issues?

Comparisons?

- How bad is this?
- Each theory is highly speculative.
- Are there any theories we trust with similar issues?
- **QED!** (Drummond and Hathrell, 1979)
- Similar effect occurs for *photon* propagation in QED near black holes.
- Due to e^{-} induced non-minimal photon-gravity couplings.

The Drummond-Hathrell EFT

$$\mathcal{L} = M_{pl}^2 R - \frac{1}{4e^2} F_{\mu\nu}^2 + \bar{\psi}\left(i\not{D} - m\right)\psi$$



$$\mathcal{L} = M_{pl}^2 R - \frac{1}{4e^2} F_{\mu\nu}^2 + \frac{c_1}{m^4} F_{\mu\nu}^4 + \frac{c_2}{m^2} R_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} + \ldots$$

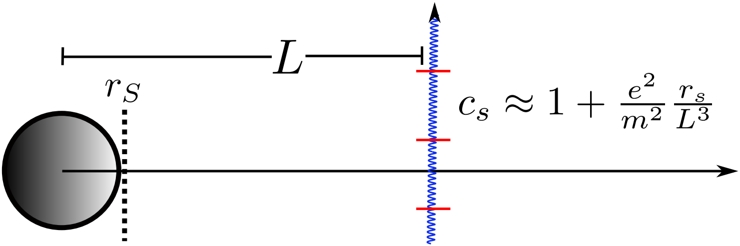
- If e^-'s aren't important, work with the EFT.
- Any term with $F_{\mu\nu}$ can alter photon propagation.
- Different behaviors arise on different backgrounds.

Ex: Constant Magnetic Fields (Adler, 1971)

$$\mathcal{L} = -\frac{1}{4e^2}F_{\mu\nu}^2 + \frac{7}{90(4\pi)^2 m^4}F^{\mu}{}_{\nu}F^{\nu}{}_{\rho}F^{\rho}{}_{\sigma}F^{\sigma}{}_{\mu} - \frac{1}{36(4\pi)^2 m^4}\left(F_{\mu\nu}F^{\mu\nu}\right)^2 + \ldots$$

- For example, photons travel more slowly in strong magnetic fields.
- Geometric optics: $c_s \sim 1 - e^4 B^4/m^4$. Lightcone narrows, $\tilde{g}_{\mu\nu} \neq \eta_{\mu\nu}$.
- $\mathcal{O}(10\%)$ effect in pulsars.
- Virtual electrons act as an effective medium. *Vacuum effect*.
- Similar conclusions hold for arbitrary EM backgrounds (Daniels and Shore, 1993).

The Drummond-Hathrell Problem (1979)



$$\mathcal{L} = M_{pl}^2 R - \frac{1}{4e^2}F_{\mu\nu}^2 + \frac{c_2}{m^2}R_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma} + \ldots$$

- *Same* analysis: Photon is "superluminal" near black holes.
- Lightcone widens: $\tilde{g}_{\mu\nu} \approx \bar{g}_{\mu\nu} + \frac{8c_2e^2}{m^2}\bar{R}_{\mu\rho\nu\sigma}f^\rho f^\sigma$.
- Occurs for *radially* polarized γ propagating in *angular* directions.
- Other polarization is subluminal. Radially propagating γ's are luminal.

EFT Artifact?

- Longstanding oddity. Many have looked into this, varying conclusions.
- (Shore, Hollowood) prominent, explore high energy limit.
- Intuition: QED superluminality should be an artifact of EFT.
- How can this be fake, while constant B case is real?
- We'd like a detailed understanding of how QED "protects" itself.
- Important to understand all approximations made in EFT.

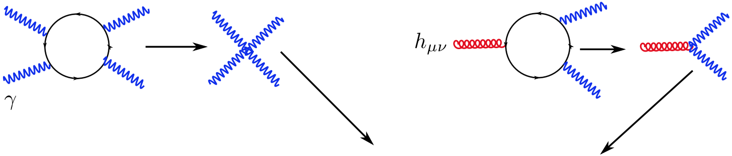
Effective Field Theory (EFT) and QED

- General idea: mimic short distance physics by an effective description.
- Our case: e^{-}'s not so relevant for γ propagation, BHs.
- Remove them. Fewer fields: just A_μ and $g_{\mu\nu}$.
- Technically easier to work with effective description.
- Allows for efficient approximation scheme.

Building an EFT: Matching

- How do we build the EFT?
- Method 1: Match calculations in full and effective theories.

$$\mathcal{L} = M_{pl}^2 R - \frac{1}{4e^2} F_{\mu\nu}^2 + \bar{\psi}\left(i\not{D} - m\right)\psi$$



$$\mathcal{L} = M_{pl}^2 R - \frac{1}{4e^2} F_{\mu\nu}^2 + \frac{c_1}{m^4} F_{\mu\nu}^4 + \frac{c_2}{m^2} R_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} + \dots$$

- Light by light scattering famous example. *RFF* more relevant for us.

Building an EFT: Integrating Out

- Method 2: Integrate out the e^-.

$$\exp iS_{\rm EFT}[A_\mu, g_{\mu\nu}] = \int \mathcal{D}\psi\mathcal{D}\bar{\psi}\, \exp iS_{\rm QED}[A_\mu, g_{\mu\nu}, \psi, \bar{\psi}]$$

- QED ideal for functional methods $S_{\rm EFT} \supset {\rm Tr}\ln\left(i\not{D} - m\right)$.

- In principle, this just splits the calculation into two steps.

$$\begin{aligned}\langle A_\mu(x)A_\nu(y)\rangle &= \int \mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}A_\mu\mathcal{D}g_{\mu\nu}\, e^{iS_{\rm QED}} A_\mu(x)A_\nu(y) \\ &= \int \mathcal{D}A_\mu\mathcal{D}g_{\mu\nu}\, e^{iS_{\rm EFT}} A_\mu(x)A_\nu(y)\end{aligned}$$

- No information would be lost if we could do the above path integral.

- However, we can't. Necessarily make approximations.

EFT: Approximations and Validity

- Can't keep all terms in $S_{\rm EFT}$. Must truncate.

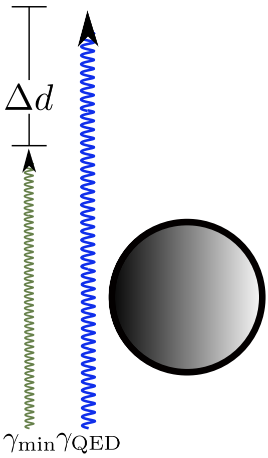
$$\mathcal{L} = M_{pl}^2 R - \frac{1}{4e^2} F_{\mu\nu}^2 + \frac{c_1}{m^4} F_{\mu\nu}^4 + \frac{c_2}{m^2} R_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} + \dots$$

- Drop terms higher order in R/m^2, F/m^2.
- Setup must keep these terms small. Otherwise, EFT is invalid.
- Energies, curvatures, field strengths $\ll m$. Lengths $\gg m^{-1}$.

EFT Criteria for Superluminality: QED

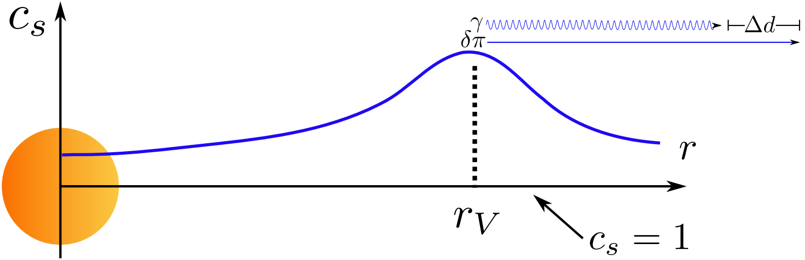
$$-\frac{1}{4e^2}F_{\mu\nu}^2 + \frac{c_2}{m^2}R_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$$

- Natural criteria: compare distance advance to m^{-1}.
- Race a *minimally* coupled photon against QED photon.
- Solve geodesic equation for $\tilde{g}_{\mu\nu}$.
- Shapiro delay cancels.
- $\Delta d \lesssim m^{-1}\left(\frac{e^2}{mr_s}\right) \ll m^{-1}$. *Tiny*.
- $\Delta d \sim 10^{-31}$ meters for e^-, $M_\odot$ BH.



EFT Criteria for Superluminality: Galileons

- Cubic Galileon: $\mathcal{L} = -\frac{1}{2}(\partial\pi)^2 - \frac{1}{\Lambda^3}(\partial\pi)^2\Box\pi + \frac{\pi}{M_{pl}} T^{\mu}{}_{\mu}$.



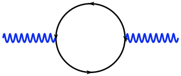
- Now, Λ^{-1} is the cutoff. Same role as m^{-1} in QED.
- Race $\delta\pi$ against a photon from r_V to infinity.
- Macroscopic superluminality $\Delta d \sim r_V \sim \Lambda^{-1} \left(M/M_{pl}\right)^{1/3} \gg \Lambda^{-1}$.

Recap and Other Setups

- QED, modified GR qualitatively different in simplest scenario.
- $\Delta d_{\rm QED} \ll m^{-1}$ while $\Delta d_{\rm galileon} \gg \Lambda^{-1}$.
- QED superluminality not "real", apparently. Similar to EFT ghosts.
- What about other QED setups?
- Let's go to extremes, see what happens.
- Issues should be resolved *within* EFT. Stick to low energies.

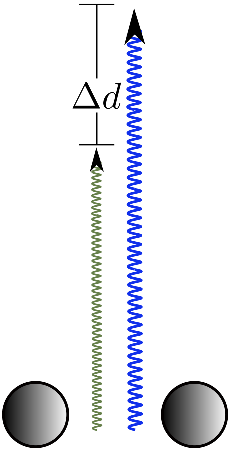
Two Failed Attempts: Small black holes and large N_f.

- QED photon wins by $\Delta d \lesssim m^{-1}\left(\frac{e^2}{mr_s}\right)$.
- Tiny black holes:
  - Make denominator small, $r_s \ll m^{-1}$.
  - But, curvatures $\mathcal{O}(1/r_s^2)$, $\implies R_{\mu\nu\rho\sigma}/m^2 \gg 1$. EFT breaks down.
- Large number of species, N_f:
  - Now, $\Delta d \approx m^{-1}\left(\frac{N_f e^2}{mr_s}\right)$.
  - Make numerator large, $N_f e^2 \gg 1$.
  - Physics becomes non-perturbative, can't calculate.

 $\sim N_f e^2$

A Better Attempt: Many Black Holes

$$\Delta d \approx m^{-1} N_{\rm BH} \left(\frac{e^2}{m r_s}\right)$$



- Amplify using *many* black holes.
- This setup is our main focus.
- Pairs of black holes prevent curving.
- Note: Absurd. Shows how hard $\Delta d > m^{-1}$ is.
- $N_{\rm BH} \sim \frac{m r_s}{e^2} \sim 10^{17}$ for e^-, $M_\odot$ BH.

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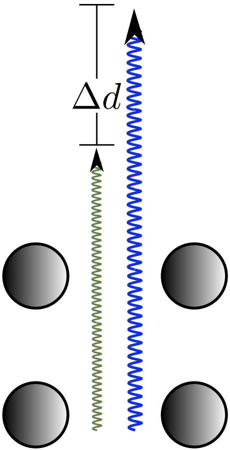

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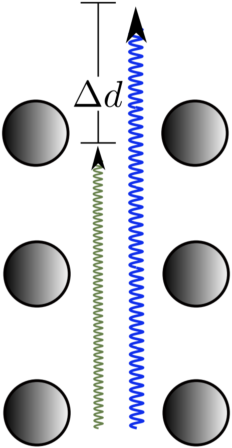

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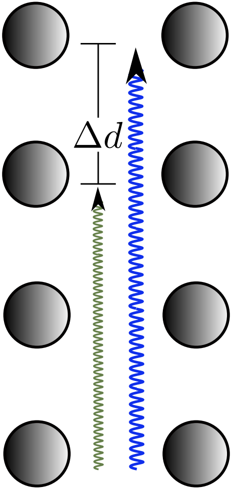

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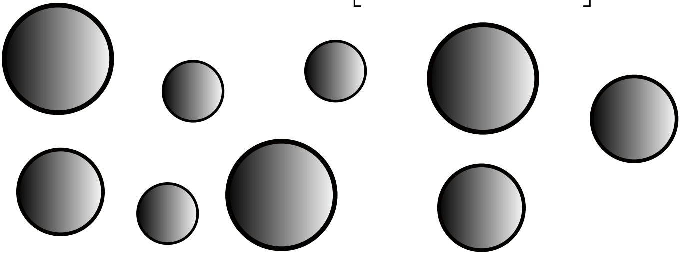
Preventing Collapse: Majumdar-Papapetrou Solutions

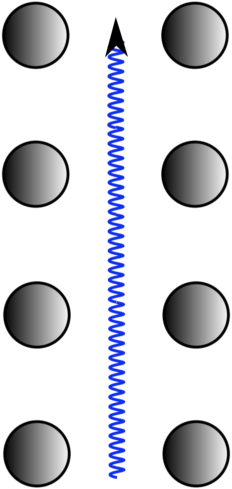
- We need to stop ladder from collapsing.
- Use many charged, extremal Reissner-Nordstrom black holes.

$$S = \int \mathrm{d}^4x\sqrt{-g}\,\left[M_{pl}^2 R - \frac{1}{4e^2}F_{\mu\nu}^2\right]$$



- An exact, classical solution of *pure* Einstein-Maxwell (no e^-'s!).
- GR attraction and EM repulsion perfectly balanced, $Q = M/M_{pl}\sqrt{2}$.

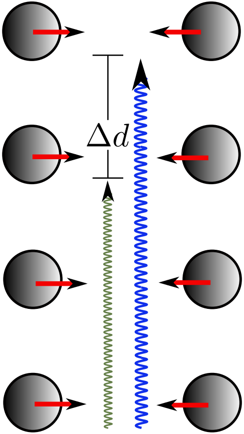
Preventing Collapse?



- *If* tunnel is stable, unbounded superluminality.
- Seems crazy.
- What happens?

The Punchline: Collapse Just In Time

$\Delta d_{\max} \approx e \times m^{-1}$



- No longer an exact solution with e^-'s.
  $$\mathcal{L} = M_{pl}^2 R - \frac{1}{4e^2} F_{\mu\nu}^2 + \frac{c_2}{m^2} R_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} \ldots$$
- Background forces cancel.
- But, corrections destabilize setup: *collapse*.
- Tunnel collapses before $\Delta d > m^{-1}$ achieved.
- Lots of interesting physics in details.

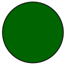
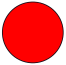
Finding Perturbative BH Solutions (Duff, 1973)

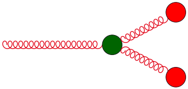
- Goal: Find perturbative corrections to $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$.

- Feynman diagrams are excellent for this purpose.

- Allow for easy estimates of contributions. E.g. Schwarzschild:

$$S \sim \int \mathrm{d}^4x \, M_{pl}^2 (\partial h)^2 + M_{pl}^2 h (\partial h)^2 + \ldots - M \int \mathrm{d}\tau$$

 $\sim \dfrac{1}{M_{pl}^2}$  $\sim M_{pl}^2$  $\sim M$

$h_{\mu\nu}$:  $\sim \dfrac{M}{M_{pl}^2 r} \sim \dfrac{r_s}{r}$ $+$  $\sim \left(\dfrac{r_s}{r}\right)^2$ $+ \ldots$

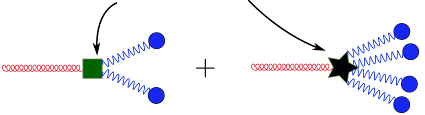
- Summing tree diagrams $\leftrightarrow$ solving EOM perturbatively.

Keeping Corrections Small

- Diagrams help ensure corrections are small. Keep us within EFT.

$$S = \int \mathrm{d}^4x \sqrt{-g}\, \left(-\tfrac{1}{4e^2} F_{\mu\nu}^2 + \tfrac{c_1}{m^4} F_{\mu\nu}^4 + \ldots\right) + \tfrac{Q}{e} \int \mathrm{d}x^\mu A_\mu$$

$h_{\mu\nu}$:



$+ \quad \cdots$

- Keeping right diagram small $\implies r_s \gg m^{-1}\left(\frac{eM_{pl}}{m}\right)$.
- Extremal RN BHs must be of minimum size to be within EFT.
- Schwinger pair production is becoming important (Gibbons, 1975).
- Funny numerology: for SM $M_{\mathrm{min}} \sim \mathcal{O}(10^5 M_\odot)$.

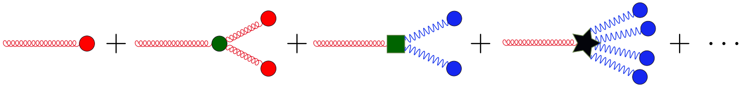
Missing Physics

- Diagrams also help avoid making mistakes by missing physics.
- For example, could solve EOM perturbatively using:

$$\mathcal{L} = M_{pl}^2 R - \frac{1}{4e^2} F_{\mu\nu}^2 + \frac{c_1}{m^4} F_{\mu\nu}^4 + \frac{c_2}{m^2} R_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} + \dots$$

- Equivalent to summing all *tree* diagrams.

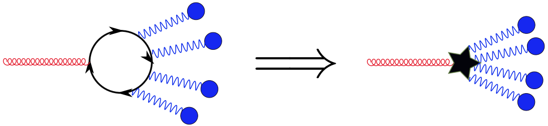
$h_{\mu\nu}$:



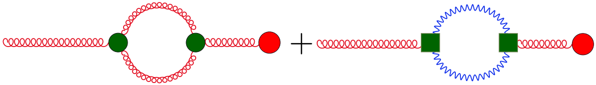
- Misses physics, gives a qualitatively *wrong* answer.

The Mistake: No Massless Loops

- Recall: EFT operators come from loops of e^-'s.



- Why don't we include photon, graviton loops, too?

$h_{\mu\nu}$:  $+ \dots$

- These *need* to be included. Give important effects.
- Expect light loops dominate at large distances.
- Diagrams make it clear these should be calculated.

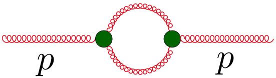
Quantum Gravity?

- Graviton loops $\implies$ quantum gravity $\implies$ scary?
- No. Low energy predictions extractable. (Duff, 1974)(Donoghue, 1993)
- GR+corrections is an entirely reasonable, low energy EFT.
- It's the *UV* completion we don't understand.

Quantum Gravity!

$$S = \int \mathrm{d}^4x\sqrt{-g}\,\left[M_{pl}^2 R + c_1 R^2 + c_2 R_{\mu\nu}^2 + \frac{c_3}{M_{pl}^2} R^3 + \ldots\right]$$

- Include all possible operators in EFT.

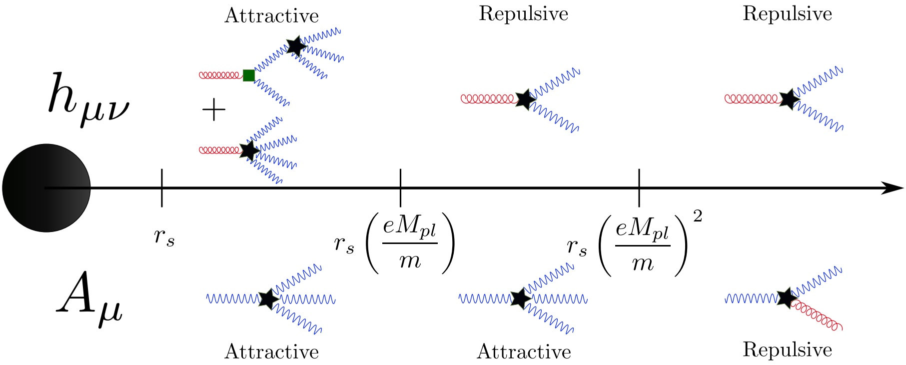


$$\Longrightarrow \; \Sigma \sim \frac{1}{\epsilon}p^4 + p^4 \ln p^2/\mu^2$$

- R^2, $R_{\mu\nu}^2$ counterterms absorb $1/\epsilon$, $\ln \mu$ determines β functions.
- No local counterterms affects $p^4 \ln p^2$. Non-analyticity the key.
- Equivalently: $p^4 \ln p^2$ bit independent of whatever UV completes GR.
- Generates $\delta g_{tt} \sim \left(\frac{r_s}{r}\right)\left(\frac{1}{M_{pl} r}\right)^2$ *attractive* potential. (Duff, Donoghue)

Necessity of Light Loops 1

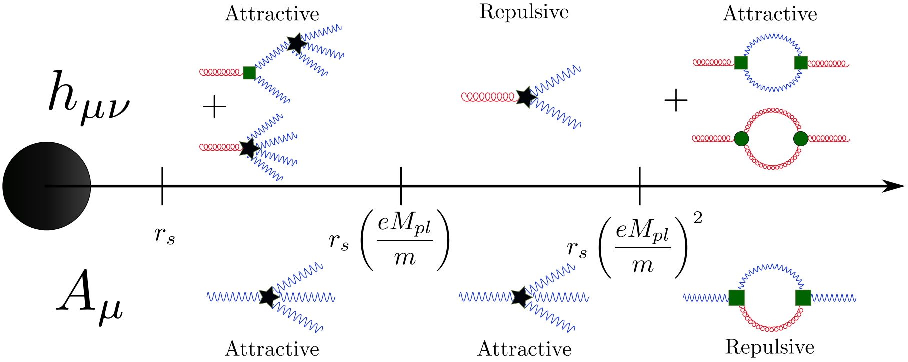
- Need the loops. Can't just solve EOM.
- Without loops, the dominant corrections are:



- *RFF* terms cause repulsion for $r \gtrsim r_s\left(\frac{eM_{pl}}{m}\right)^2$.

Necessity of Light Loops 2

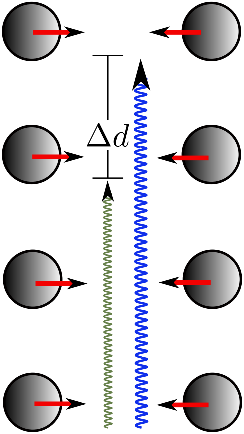
- Loops enter at *exactly* right scale to keep attraction.
- With loops, potential between BHs is attractive at all distances.



- Makes an important, qualitative difference in the behavior.

Total Time Advance

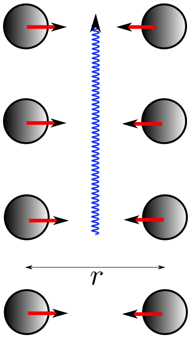
$\Delta d_{\max} \approx e \times m^{-1}$



- Back to the punchline.
- Given e^- induced potentials, calculate forces.
- Evaluate $\gamma_{\rm QED}$'s propagation speed along path.
- Summing up: $\Delta d_{\max} \approx e \times m^{-1} < m^{-1}$.
- No macroscopic superluminality.

Quick Sketch

$\Delta d_{\max} \approx e \times m^{-1}$



- Photon's speed: $\delta c_s \sim \frac{e^2}{m^2} R_{\mu\nu\rho\sigma} \sim \frac{e^2}{m^2} \frac{r_s}{r^3}$
- All $\sim (F_{\mu\nu})^n$ effects on c_s cancel by symmetries.
- Focus on $r \gtrsim r_s \left(\frac{eM_{pl}}{m}\right)^2$ where light loops dominate.
- Here, $\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2 \sim \frac{r_s}{r} \left(\frac{1}{M_{pl} r}\right)^2$
- $\Delta d \sim \int_0^{t_f} \mathrm{d}t\, \delta c_s \sim e^2 \frac{M_{pl}}{m^2} \sqrt{\frac{r_s}{r}}\Big|_{\infty}^{r_s (eM_{pl}/m)^2} \sim e \times m^{-1}$

QED Summary & Galileons/DGP/mGR

- One black hole only leads to tiny superluminality, $\Delta d \lll m^{-1}$.
- Highly elaborate, contrived construction needed to amplify effect.
- Despite efforts, never achieved $\Delta d > m^{-1}$. Parametrically smaller.
- Very non-trivial conspiracy. Supports m^{-1} as correct measure.
- QED and modified GR qualitatively different, apparently.

Other Possibilities: Overcharging

- Can also work with new features of QED black holes.
- Find near horizon $\mathcal{O}(\hbar)$ corrections to all orders in r_s/r.
- For example, for previously extremal RN black hole ($\lambda \equiv eM_{pl}/m$):

$$g_{tt} = -\Delta + \frac{\lambda^4 l_p^2 r_s^4 \hbar}{7200\pi^2 r^6} + \frac{\lambda^2 l_p^2 r_s^4 \hbar}{9600\pi^2 r^6} + \frac{\lambda^2 l_p^2 r_s^3 \hbar}{2880\pi^2 r^5} - \frac{\lambda^2 l_p^2 r_s^2 \hbar}{360\pi^2 r^4}$$

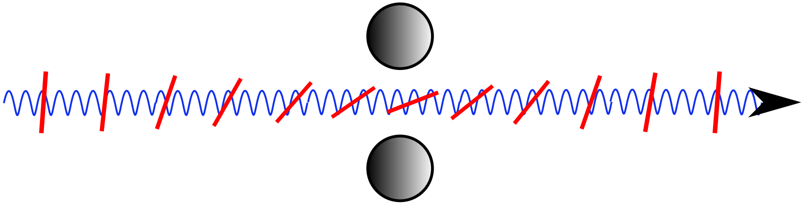
$$g_{rr} = \Delta^{-1} + \Delta^{-2}\left[\frac{\lambda^4 l_p^2 r_s^4 \hbar}{7200\pi^2 r^6} - \frac{13\lambda^2 l_p^2 r_s^4 \hbar}{7200\pi^2 r^6} + \frac{23\lambda^2 l_p^2 r_s^3 \hbar}{2880\pi^2 r^5} - \frac{\lambda^2 l_p^2 r_s^2 \hbar}{96\pi^2 r^4}\right]$$

$$\lambda \equiv \frac{eM_{pl}}{m}\,, \quad \Delta = (1 - r_s/2r)^2$$

- New feature: Black holes can carry more charge
$Q \leq \frac{M}{\sqrt{2}M_p} + \frac{8\sqrt{2}\lambda^4 M_p \hbar}{225M} - \frac{8\sqrt{2}\lambda^2 M_p \hbar}{75M}$.
- Attempts to balance forces still generate $\Delta d \sim e \times m^{-1}$.

Other QED Protections: Rotating Polarizations

- More extreme setups? Infinite lattice?
- Non-minimal couplings cause polarization to rotate during flight.
- Geometric Optics: $\delta A_\mu = (a_\mu + \epsilon b_\mu + \ldots)e^{i\theta/\epsilon}$, $k_\mu \equiv \nabla_\mu \theta$.
- $\mathcal{O}(\epsilon^{-2})$: $k_\mu k_\nu \bar{g}^{\mu\nu} = 8c_2 e^2 m^{-2} R_{\mu\nu\rho\sigma} k^\mu f^\nu k^\rho f^\sigma$, $f_\mu \propto a_\mu$
- $\mathcal{O}(\epsilon^{-1})$: $k^\mu \nabla_\mu f_\nu = \Pi_\nu{}^\mu \mathcal{S}_\mu \sim \mathcal{O}\left((\frac{e}{mr_s})^2 \right)$
- Miniscule effect, but can build up. Will tend to wash out effects.



Conclusions

- Any EFT superluminality should be compared to $\Lambda_{\mathrm{EFT}}^{-1}$.
- Seems to distinguish superluminality in QED and modified GR.
- Very problematic for DGP/mGR/Galileons.
- QED protects itself from superluminality in non-trivial way.
- Great EFT application: integrating out matter, EFT of GR...
- Future: BH phenomenology, Weak Gravity Conjecture ($eM_{pl}/m > 1$), graviton propagation, etc.

Thank you!

Thank you for listening!