Giving Gravity a Mass

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Introduction

- Let's give the graviton a mass
- Why?
- How?
- What happens?

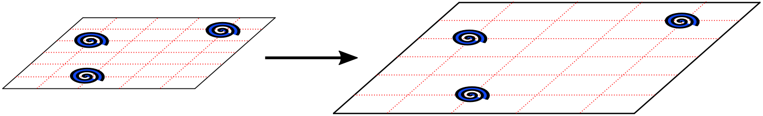
$$S = M_p^2 \int \mathrm{d}^4x \sqrt{-g} R + \text{mass?}$$

Why?

Why would we give the graviton a mass?

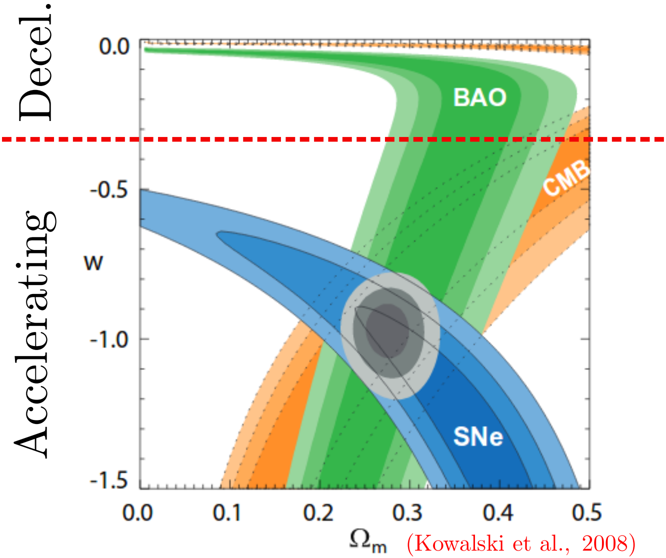
Cosmic Acceleration

- Long distances: $\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2\mathrm{d}\vec{x}^2 \quad$ (FRW)
- Scale factor $a(t)$: how space between distant points changes



- $a(t)$ determined by matter composition
- Dust: $a \propto t^{2/3}$. Radiation: $a \propto t^{1/2}$. "Normal matter" gives $\ddot{a} < 0$
- **In reality:** $\ddot{a} > 0$!

Cosmic Acceleration: Data



(Kowalski et al., 2008)

What Could It Be?

- Einstein's equation governs $a(t)$ evolution

Dynamics

Matter Content

$$M_p^2 G_{\mu\nu} = T_{\mu\nu}$$

Derivatives of $a(t)$

% Dust, radiation . . .

- Need to change something to get acceleration, $\ddot{a} > 0$

What Could It Be? A Minimal Solution

- For acceleration, add a constant: Λ $\quad a(t) \sim \exp \frac{\sqrt{\Lambda}}{M_p} t$

Dynamics

Matter Content

$$\Lambda g_{\mu\nu} + M_p^2 G_{\mu\nu} = T_{\mu\nu}$$

Derivatives of $a(t)$

% Dust, radiation . . .

- A non-diluting, constant, eternal source of acceleration.

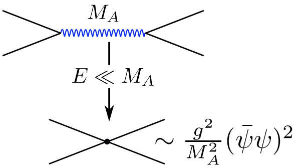
Effective Field Theories

- Good reason to expect Λ: EFT
- Idea: High energy physics captured by low energy Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu} - \frac{M_A^2}{2}A_\mu^2 + \bar{\psi}\left(i\partial\!\!\!/ + gA\!\!\!/\right)\psi + \dots$$

$$\Big\downarrow \ \text{Low Energies}$$

$$\mathcal{L} \sim i\bar{\psi}\partial\!\!\!/\psi + \frac{c_1}{M_A^2}\left(\bar{\psi}\psi\right)^2 + \frac{c_2}{M_A^5}\left(\bar{\psi}\psi\right)^3 + \dots$$



- Result: Low energy theory of ψ, dimensions fixed by $E \sim M_A$
- Very general story. "Everything not forbidden is allowed"

EFT Guidelines

- Modern viewpoint: most QFT's are EFT's

Effective Field Theory Rules:

1. List all fields and their symmetries
2. Write action with all compatible terms
3. Fix dimensions using energy scale of E. $E \sim$ UV physics
4. Expect: numerical factors $\sim \mathcal{O}(1)$

$$\mathcal{L}_{\rm UV} = \mathcal{L}(\partial\phi, \psi, \ldots), \qquad \phi \to \phi + c$$

$$\Big\downarrow \text{ Low Energies}$$

$$\mathcal{L}_{\rm IR} = -\tfrac{1}{2}(\partial\phi)^2 + \tfrac{c_1}{E^4}(\partial\phi)^4 + \tfrac{c_1}{E^8}(\partial\phi)^6 + \ldots$$

Worry for Gravity

$$S_{UV} = \int \mathrm{d}^4x\sqrt{-g}\mathcal{L}(g_{\mu\nu}, R_{\mu\nu}, \dots, \psi)$$

$$\Big\downarrow \text{ Low Energies}$$

$$S_{IR} = \int \mathrm{d}^4x\sqrt{-g}\left(\underbrace{c_0 E^4}_{\Lambda} + \underbrace{c_1 E^2}_{M_p^2} R + c_2 R^2 + c_3 R^3/E^2 + \dots\right)$$

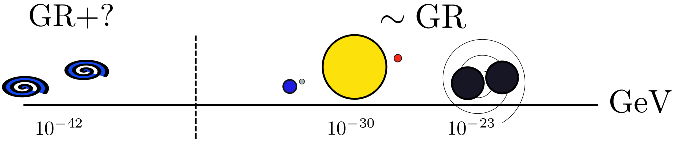
$$c_0 E^4 \to g_{\mu\nu}, \quad c_1 E^2 R \to G_{\mu\nu}$$

- **Problem**: Doesn't seem to work for gravity. Λ *tiny*, not generic
- Expect all scales to be roughly similar. Or at least $\Lambda \sim E^4_{\text{particle physics}}$
- Instead $\Lambda \sim 10^{-47}\ \text{GeV}^4$. $m^4_{\text{electron}} \sim 10^{-14}\ \text{GeV}^4$, $M_p^4 \sim 10^{73}\ \text{GeV}^4$
- Selection bias? We don't exist if Λ too large (Weinberg, 1989)

Something else?

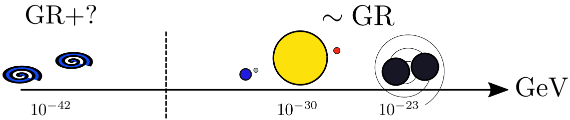
- Something more drastic?

$$M_p^2 G_{\mu\nu} + ? = T_{\mu\nu}$$



- The challenge: Modify low energy physics while leaving higher energy physics unchanged (solar system tests, LIGO results...)
- A different question than usual: typically changing UV
- Opportunity to tinker with GR.
- **Learn about gravity by seeing what changes and breaks**

Adding a Mass: a Natural Deformation



- Mass term: simplest way to change IR. Irrelevant at $E \gg m$
- Doesn't require more fields: $\Delta\mathcal{L} \sim m^2 h^2$
- A mass changes how far a field can propagate. Long distance mod.
- Ex. Yukawa potential

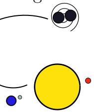
$$\mathcal{L} = -\tfrac{1}{2}(\partial\phi)^2 + \phi J \quad \Longrightarrow \quad V \sim \frac{1}{r}$$

$$\mathcal{L} = -\tfrac{1}{2}(\partial\phi)^2 - \frac{m^2}{2}\phi^2 + \phi J \quad \Longrightarrow \quad V \sim \frac{e^{-mr}}{r}$$

Degravitation via Mass?

- Massive graviton could realize following intriguing idea:
- A high-pass filter for gravity (Arkani-Hamed et al.,2002)

$$\overset{\text{scale dependent?}}{M_p^2(k)}G_{\mu\nu} = \overset{\text{long wavelength}}{-\Lambda g_{\mu\nu}} + \overset{\text{short wavelength}}{T_{\mu\nu}}$$



- Λ large as expected, but doesn't feed curvature in naive way
- Massive graviton can't propagate far enough to "see" Λ
- Works for spin-1. Mass filters constant charge background (Dvali et al.,2007)

QFT Motivation: Gravitational Higgs Mechanism?

- Understanding massive spin-1 was one of the great scientific advancements of the 20th century
- Higgs mechanism a cornerstone of the Standard Model
- Worth exploring the natural generalization to spin-2 (gravity)
- What is possible within field theory?

Massive GR: Theoretical History

- Fierz & Pauli first to write down unique spin-2 mass term (1939)

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- EFT treatment clarified all issues (Arkani-Hamed et al., 2003)
- Fully non-linear theory found in 2010 (de Rham et al., 2010)

How?

How do we add a mass to gravity?

How?

How do we add a mass to gravity?

(Such that $m \to 0 \implies \text{GR}$)

What mass term?

- Try to add mass
- Around flat space: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}/M_p$. Don't worry about Λ yet

$$\mathcal{L} = \frac{1}{2}(\partial h)^2 - \partial^\mu h \partial^\nu h_{\mu\nu} + \partial_\nu h_{\mu\rho}\partial^\rho h^{\mu\nu} - \frac{1}{2}\partial_\mu h_{\nu\rho}\partial^\mu h^{\nu\rho}$$

- Mass term isn't obvious. Two different structures

$$\mathcal{L}_{\text{mass}} = \frac{m^2}{2}\left(a_1 h_{\mu\nu}h^{\mu\nu} + a_2 h^2\right), \qquad h \equiv h_{\mu\nu}\eta^{\mu\nu}$$

- Breaks gauge-invariance (diffeomorphisms): $h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$
- What sets the tuning? Clean way to see?

Stuckelberging: Spin-1

- Massive vector example is simpler
- Restoring gauge-invariance clarifies everything

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - a_1 \times \frac{m^2}{2}A_\mu^2 \,, \qquad A_\mu \cancel{\to} A_\mu + \partial_\mu \xi$$

- Let $A_\mu = \tilde{A}_\mu + \frac{1}{m}\partial_\mu \pi$. Now: $\delta\tilde{A}_\mu = \partial_\mu \xi$, $\delta\pi = -m\xi$ symmetry
- $\tilde{A}_\mu \sim 2$ vector modes, $\pi \sim 1$ longitudinal scalar mode ($\sim$ Goldstone)

$$\mathcal{L} \supset -\frac{a_1}{2}(\partial\pi)^2$$

- a_1 simply determined by stability: $a_1 > 0$

Stuckelberging: Spin-1 Interactions with Matter

- Couple to conserved source:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{m^2}{2}A_\mu^2 + A_\mu J^\mu$$

- Stuckelberg: π decouples from matter

$$\mathcal{L} \supset -\frac{1}{4}\tilde{F}_{\mu\nu}^2 - \frac{m^2}{2}\tilde{A}_\mu^2 + \tilde{A}_\mu J^\mu$$
$$-\frac{1}{2}(\partial\pi)^2 - \cancel{\frac{1}{m}\pi\partial_\mu J^\mu} \to 0$$

- Mass is a mild deformation. Physics is continuous as $m \to 0$.

Stuckelberging: Spin-2

$$\mathcal{L}_{\rm mass} = \frac{m^2}{2}\left(a_1 h_{\mu\nu}h^{\mu\nu} + a_2 h^2\right)$$

- Spin-2: same trick invaluable (Arkani-Hamed, 2002)

$$h_{\mu\nu} = \tilde{h}_{\mu\nu} + \frac{1}{m}\partial_{(\mu}A_{\nu)} + \frac{1}{m^2}\partial_\mu\partial_\nu\pi$$

$$\mathcal{L} \supset \frac{2(a_1+a_2)}{m^2}(\Box\pi)^2 + 2(a_1+a_2)(\partial_\mu A^\mu)^2 + (a_1-a_2)F_{\mu\nu}^2$$

- Want 5 DOF: 2 tensor $\tilde{h}_{\mu\nu}$, 2 vector A_μ, 1 scalar π
- Stability/DOF: $a_1 + a_2 = 0$, $a_1 - a_2 < 0$. Fierz-Pauli mass term (1939)

$$\mathcal{L}^{\rm FP}_{\rm mass} = \frac{m^2}{2}\left(-h_{\mu\nu}h^{\mu\nu} + h^2\right)$$

Stuckelberging: Spin-2

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$$\mathcal{L} \supset \cancel{\frac{2(a_1+a_2)}{m^2}(\Box\pi)^2} + \cancel{2(a_1+a_2)(\partial_\mu A^\mu)^2} + (\overbrace{a_1-a_2}^{<0})F_{\mu\nu}^2$$

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$$\mathcal{L}_{\text{mass}}^{\text{FP}} = \frac{m^2}{2}\left(-h_{\mu\nu}h^{\mu\nu} + h^2\right)$$

Issue with Higher Derivatives: Ghosts

- What was wrong with having $(\Box\pi)^2$?
- Too many degrees of freedom *and* unstable
- Equivalent to two fields!

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\pi\Box\pi + \frac{\lambda}{M^2}(\Box\pi)^2 \\ &= \frac{1}{2}\pi\Box\pi + \psi\Box\pi - \frac{M^2}{4\lambda}\psi^2\end{aligned}$$

- Redefining $\pi \to \pi' + \psi$ reveals *ghost*. Add interactions $\implies$ disaster

$$\mathcal{L} = +\frac{1}{2}\pi'\Box\pi' - \psi\Box\psi - \frac{M^2}{4\lambda}\psi^2$$

- Similar interactions $\sim (\partial^2\pi)^n$ also problematic

Stuckelberging: Spin-2 Interactions With Matter

$$\mathcal{L} \supset \frac{1}{2}(\partial\tilde{h})^2 + \ldots + \frac{m^2}{2}\tilde{h}^2 + \tilde{h}_{\mu\nu}T^{\mu\nu}/M_p$$
$$+\text{Decoupled Vector} - \frac{1}{2}(\partial\pi)^2 + \pi T^{\mu}_{\mu}/M_p$$

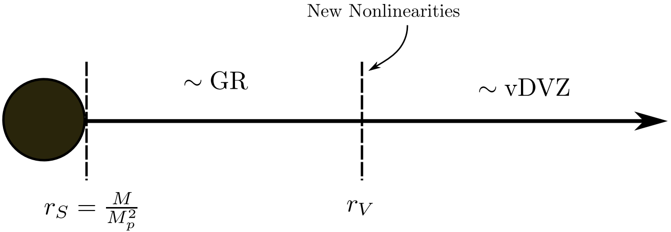
- Interactions much stranger than spin-1. π doesn't decouple
- $T^{\mu}_{\mu} = 0$ for light, $T^{\mu}_{\mu} \neq 0$ for other matter. Extra forces
- $\mathcal{O}(1)$ difference to orbits or light bending. Unacceptable
- Odd sort of discontinuity, known as vDVZ (van Dam et al., 1970)
- Haven't constructed desired theory yet. No GR as $m \to 0$

Continuity From Non-Linearities?

- If continuity with GR possible, need to tinker even *more*
- Vainshtein: non-linearities may save the day (Vainshtein, 1972)

$$\mathcal{L} = \overbrace{\frac{1}{2}(\partial h)^2 + (h/Mp)^n\,(\partial h)^2}^{R} + \overbrace{\frac{m^2}{2}\left(-h_{\mu\nu}^2 + h^2\right)}^{\mathrm{FP}} \overset{\mathrm{New}}{+a_3 h^3 + a_4 h^4 + \dots}$$

- vDVZ just the linear approximation



Adding Interactions

- Rules for adding interactions?
- Add $(h_{\mu\nu})^n$, Stuckelberg and introduce $\tilde{h}_{\mu\nu}, A_\mu, \pi$
- Generically, find terms $\sim (\partial^2\pi)^n$, higher order EOM
- Give wrong DOF count (Boulware-Deser ghost) and EFT breakdown

$$\mathcal{L}_{\rm new} = a_3 h^3 + a_4 h^4 + a_5 h^5 + a_6 h^6 + \ldots$$

- Solve order by order in h, avoiding $\sim (\partial^2\pi)^n$

dRGT Massive Gravity

- Three possible interactions ($d = 4$) (de Rham et al., 2010)
- Involves curious matrix square root structure. Complicated

$$\text{E.g. } \mathcal{L} = c_1 \left(\mathcal{K}^2_{\mu\nu} - \mathcal{K}^2\right) + c_2 \left(\mathcal{K}^3 - 3\mathcal{K}\mathcal{K}^2_{\mu\nu} + 2\mathcal{K}^3_{\mu\nu}\right) + \dots$$

$$\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \sqrt{g^{\mu\rho}\eta_{\nu\rho}}\,, \qquad g^{\mu\rho} = \eta^{\mu\nu} - h^{\mu\nu} + \mathcal{O}(h^2)$$

- Intriguingly simple in terms of vierbeins (Hinterbichler et al., 2012)

$$\mathcal{L} = \epsilon_{abcd}\Big(a_1 \mathbf{e}^a \wedge \mathbf{e}^b \wedge \mathbf{e}^c \wedge \mathbf{1}^d \wedge + a_2 \mathbf{e}^a \wedge \mathbf{e}^b \wedge \mathbf{1}^c \wedge \mathbf{1}^d + \dots\Big)$$

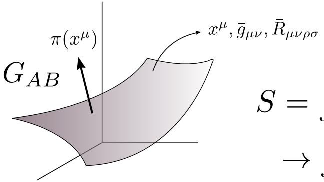
- Easily generalizes to multiple interacting spin-2's

π-Sector and Galileons

$$\mathcal{L} = -\frac{1}{2}(\partial\pi)^2 - \frac{1}{\Lambda^3}(\partial\pi)^2\Box\pi + \frac{1}{\Lambda^6}\left((\partial\pi)^2(\Box\pi)^2 - (\partial\pi)^2(\partial_\mu\partial_\nu\pi)^2\right) + \ldots$$

- Lots of fascinating QFT structure in the π-sector
- A *Galileon* theory: $\pi \to \pi + c + b_\mu x^\mu$ (Nicolis et al., 2008)
- Five terms with fewer ∂'s per π. Compare to $(\partial\partial\pi)^n$
- Lots of derivatives, but avoids the ghosts

π-Sector: Brane Construction



$$S = \int d^4x \sqrt{-\bar{g}} \left[c_1 + c_2 K + c_3 \bar{R} + \ldots \right]$$
$$\rightarrow \int \mathrm{d}^4x \; -\frac{1}{2}(\partial\pi)^2 - \frac{1}{\Lambda}(\partial\pi)^2 \Box\pi + \ldots$$

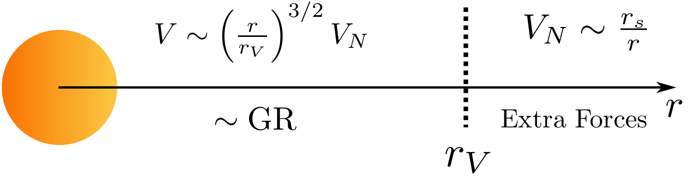
- π also appears in entirely different brane context (de Rham et al., 2010)
- $\pi \rightarrow \pi + c + b_\mu x^\mu$ corresponds to translations/boosts of brane
- Special operators $\longleftrightarrow$ Lovelock terms
- Galileons one in class of theories with similar properties (GG et al., 2011)
- Related construction as Goldstones of spacetime SSB (GG et al., 2012)

What happens?

What happens when we add a mass to GR?

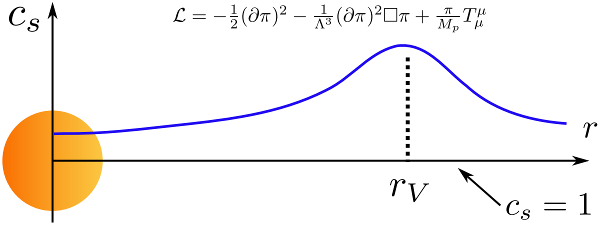
Continuity with GR: Vainshtein Screening

$$\mathcal{L} \supset -\frac{1}{2}(\partial\pi)^2 - \frac{1}{\Lambda^3}(\partial\pi)^2\Box\pi + \ldots + \frac{\pi}{M_{pl}}T^{\mu}{}_{\mu}\,, \quad \Lambda = (M_p m^2)^{1/3}$$



- 5th force suppressed via nonlinearities for $r \ll r_V = \Lambda^{-1}\left(M/M_{pl}\right)^{1/3}$.
- Continuity with GR: $r_V \to \infty$ as $m \to 0$
- $m \to 0$ makes nonlinearity *more* important
- For us, $\Lambda^{-1} \sim 10^3$km, $r_V^{\odot} \sim 10^{15}$km ($\gg r_{\text{Solar System}} \sim 10^9$km).

π Superluminality



- π-sector also where problems are
- Screening non-linearities also induces superluminality.
- $c_s > 1$ also occurs in QED(Drummond et al., 1980), but different type (GG et al., 2016)
- Full mGR analysis: superluminality for some parameters (Camanho, 2016)

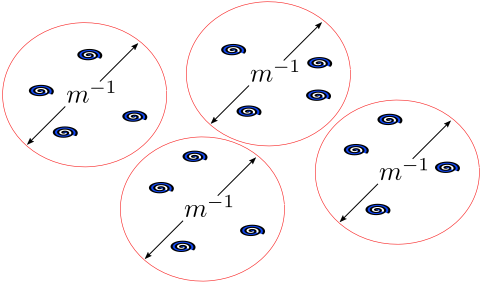
Cosmic Acceleration

- Degravitation partially works (de Rham et al., 2010)
- Given Λ, flat space a solution for appropriate mass terms.
- But mass terms then need tuning; shifts the problem
- Dynamic degravitation? Phase transitions?

$$M_p^2 G_{\mu\nu} = \cancel{-\Lambda g_{\mu\nu} + \mathcal{E}^m_{\mu\nu}}$$

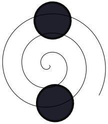
More Cosmology

- Odd cosmology
- No flat FRW solutions(D'Amico et al., 2011)
- Instead, isotropic and homogeneous regions of size $\sim m^{-1}$
- Inhomogeneities develop below certain density



Bounding m_{graviton}

- For cosmology, $m \sim H \sim 10^{-42}\mathrm{GeV}$
- Many bounds model dependent, don't apply to dRGT
- E.g. $V \sim \frac{e^{-mr}}{r}$, Mercury precession $\Longrightarrow m \lesssim 10^{-31}\mathrm{GeV}$ (Talmadge et al., 1988)
- Others do: $\omega^2 = k^2 + m^2$, grav. wave speed depends on wavelength
- LIGO: $m \lesssim 10^{-32}\mathrm{GeV}$ (Abbott et al., 2017) LISA: $m \lesssim 10^{-35}\mathrm{GeV}$

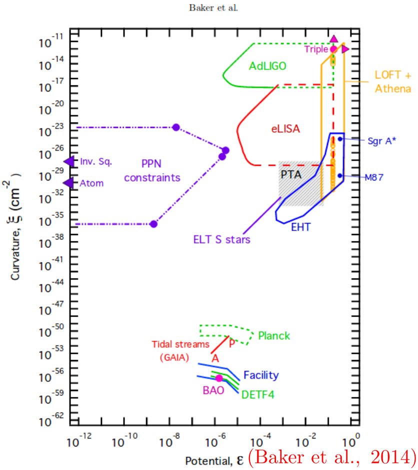


$\Delta t_{\mathrm{emission}}$

$\xrightarrow{\text{large } k \text{ faster than small } k}$

$\Delta t_{\mathrm{arrival}} < \Delta t_{\mathrm{emission}}$

One Piece in the Puzzle?



(Baker et al., 2014)

- Enormous efforts underway to nail down structure of gravity
- Theoretical guidance important to know what to look for
- E.g. $V \sim \frac{e^{-mr}}{r}$ interesting, but not a massive graviton
- Massive GR important role as one of better motivated alternatives

Conclusions

- Cosmic acceleration has motivated study of GR modifcations
- Massive GR a conservative change, in some ways (radical in others)
- Interesting QFT problem in itself to consistently add mass
- Highly nontrivial phenomenology, continuity with GR
- mGR an important benchmark in class of alternatives

Thank you!

Thank you for listening!